Explaining Educational Attainment across Countries and over Time

D. Restuccia¹ G. Vandenbroucke²

¹University of Toronto

²University of Southern California

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Introduction Model Calibration Experiment Conclusion Extra Materia

Facts: Schooling across Countries, 1950–2005

- 1. Schooling differences across countries are "large"
- 2. Schooling increased in all countries
- 3. Schooling differences smaller in 2005 than in 1950
 - Schooling increased faster for poor
 - True even if poor's GDP/cap. did not catch up

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Years of Schooling and GDP per capita (U.S. = 1)

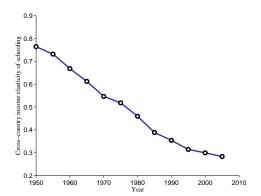
	1950		_	20	05	_
Decile	<i>y</i> 50	<i>\$</i> ₅₀		<i>y</i> 05	<i>s</i> ₀₅	s_{05}/s_{50}
1	0.05	1.28		0.06	5.01	3.91
2	0.07	1.50		0.05	6.85	4.57
3	0.09	3.18		0.21	8.42	2.65
4	0.12	2.04		0.10	7.88	3.87
5	0.17	2.43		0.22	9.41	3.87
6	0.21	3.91		0.31	9.96	2.55
7	0.24	4.06		0.34	9.95	2.45
8	0.38	5.83		0.61	11.25	1.93
9	0.58	6.70		0.71	11.75	1.75
10	0.81	7.96		0.77 11.15		1.40
$R_{10/1}$	17.56	6.22		13.95	2.23	_
$R_{9/1}$	12.51	5.23		12.85	2.35	_



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Income Elasticity of Schooling across Countries

 Holding income differences constant, the differences in years of schooling have decreased



Back

Questions

- What accounts for
 - schooling differences across countries?
 - patterns of schooling changes through time?

Back

Strategy

- A model of schooling based on Bils and Klenow (2000)
 - Exogenous productivity and life expectancy
 - ullet Non-homothetic preferences o income effect
 - Endogenous time allocation
 - Home production

Back 7/44

Strategy

- A model of schooling based on Bils and Klenow (2000)
 - Exogenous productivity and life expectancy
 - Non-homothetic preferences → income effect
 - Endogenous time allocation
 - Home production
- Fit U.S. time series of schooling and time allocation

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Strategy

- A model of schooling based on Bils and Klenow (2000)
 - Exogenous productivity and life expectancy
 - Non-homothetic preferences → income effect
 - Endogenous time allocation
 - Home production
- Fit U.S. time series of schooling and time allocation
- Cross-country experiment
 - Economies differ in Productivity Life expectancy
 - Ask

What % of schooling diff. with U.S. accounted for in 1950? What % of changes in schooling over time accounted for?

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Findings

- Cross Section:
 Model accounts for 90% of differences in schooling between
 U.S. & poor countries in 1950
- Time Series:
 Model accounts for 64% of changes in schooling over time in poor countries
- Schooling increases faster in poor economies relative to rich even if their income does not
- Emphasize role of productivity improvements for schooling in poor countries

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Model

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Environment

- Preferences: market & nonmarket cons., leisure time, schooling time
- Human capital
 - inputs of time (schooling) and goods
- Finite lives, perfect foresight, perfect credit market
- Exogenous variables
 - Productivity (market and nonmarket)
 - Life expectancy

Back 10/44

Preferences

• Lifetime utility, generation au

$$\int_{\tau}^{\tau+T_{\tau}} e^{-\rho t} \left[U(c_{\tau,t}) + \alpha V(\ell_{\tau,t}) \right] dt + \beta W(s_{\tau})$$

where

$$egin{array}{lcl} U\left(c_{ au,t}
ight) &=& \ln\left(c_{ au,t}-ar{c}
ight) \ V\left(\ell_{ au,t}
ight) &=& rac{\ell_{ au,t}^{1-\mu}-1}{1-\mu} \ W\left(s_{ au,t}
ight) &=& \ln\left(s_{ au,t}
ight) \end{array}$$

• $\bar{c} > 0$: constant

Back 11/44

Technology

• Human capital technology (from Bils and Klenow, 2000)

$$H\left(s_{\tau,t},x_{\tau,t}\right)=x_{\tau,t}^{\gamma}h\left(s_{\tau,t}\right)$$

where

$$h\left(s_{ au,t}
ight) = \exp\left(rac{ heta}{1-\psi}s_{ au,t}^{1-\psi}
ight)$$

- $x_{\tau,t}$: goods
- $s_{\tau,t}$: time (schooling)

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Technology

Household technology (from McGrattan et al., 1997)

$$c_{\tau,t} = \left[\phi\left(c_{\tau,t}^{m}\right)^{\sigma} + (1-\phi)\left(c_{\tau,t}^{n}\right)^{\sigma}\right]^{1/\sigma}$$

where

$$c_{\tau,t}^n = z_{\tau}^n n_{\tau,t}$$

- $c_{\tau,t}^m$: market goods
- $c_{\tau,t}^n$: nonmarket goods
- $n_{\tau,t}$: nonmarket, nonleisure time
- z_{τ}^{n} : household productivity

Back 13/44

Optimization

• We impose $c_{ au,t}^i=c_{ au}^i$ (i=m,n), $\ell_{ au,t}=\ell_{ au}$, and $n_{ au,t}=n_{ au}$

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Optimization

- We impose $c_{ au,t}^i=c_{ au}^i$ (i=m,n), $\ell_{ au,t}=\ell_{ au}$, and $n_{ au,t}=n_{ au}$
- Optimization problem becomes:

$$\max_{\boldsymbol{c}_{\tau}^{m},\boldsymbol{c}_{\tau}^{n},\boldsymbol{\ell}_{\tau},\boldsymbol{x}_{\tau},\boldsymbol{s}_{\tau}}\int_{0}^{T_{\tau}}e^{-\rho t}\left[U(\boldsymbol{c}_{\tau})+\alpha V(\ell_{\tau})\right]dt+\beta W\left(\boldsymbol{s}_{\tau}\right)$$

subject to

$$c_{\tau}^{m} \int_{0}^{T_{\tau}} e^{-\rho t} dt + x_{\tau} = z_{\tau}^{m} (1 - n_{\tau} - \ell_{\tau}) H(s_{\tau}, x_{\tau}) \int_{s}^{T_{\tau}} e^{(g^{m} - \rho)t} dt$$

and technologies

- z_{τ}^{m} : market productivity
- g^m : rate of growth of z_{τ}^m

Back 14/44

First Order Condition for s_{τ}

$$\underbrace{\beta W'\left(s_{\tau}\right)}_{\text{marg. benef.}} = \underbrace{-\frac{\phi}{1-\gamma} a_{\tau} U'\left(c_{\tau}\right) c_{\tau} \left(\frac{c_{\tau}^{m}}{c_{\tau}}\right)^{\sigma} \underbrace{\left[\frac{h'(s_{\tau})}{h(s_{\tau})} + \frac{d'_{\tau}(s_{\tau})}{d_{\tau}(s_{\tau})}\right]}_{\text{marg. cost}}$$

where
$$a_{ au}=\int_0^{T_{ au}}e^{-
ho u}du$$
 and $d_{ au}(s)=\int_s^{T_{ au}}e^{(g^m-
ho)u}du$

Back 15/44

First Order Condition for s_{τ}

$$\underbrace{\beta W'\left(s_{\tau}\right)}_{\text{marg. benef.}} = \underbrace{-\frac{\phi}{1-\gamma} a_{\tau} U'\left(c_{\tau}\right) c_{\tau} \left(\frac{c_{\tau}^{m}}{c_{\tau}}\right)^{\sigma} \left[\frac{h'(s_{\tau})}{h(s_{\tau})} + \frac{d'_{\tau}(s_{\tau})}{d_{\tau}(s_{\tau})}\right]}_{\text{marg. cost}}$$

where
$$a_{ au}=\int_0^{T_{ au}}e^{-
ho u}du$$
 and $d_{ au}(s)=\int_s^{T_{ au}}e^{(g^m-
ho)u}du$

- If $\beta = 0$
 - maximize lifetime income: $A_{\tau}(s_{\tau}) = 0$
 - s_{τ} independent of productivity

Back 15/44

First Order Condition for s_{τ}

$$\underbrace{\beta W'\left(s_{\tau}\right)}_{\text{marg. benef.}} = \underbrace{-\frac{\phi}{1-\gamma} a_{\tau} U'\left(c_{\tau}\right) c_{\tau} \left(\frac{c_{\tau}^{m}}{c_{\tau}}\right)^{\sigma} \underbrace{\left[\frac{h'(s_{\tau})}{h(s_{\tau})} + \frac{d'_{\tau}(s_{\tau})}{d_{\tau}(s_{\tau})}\right]}_{\text{marg. cost}}$$

where
$$a_{ au}=\int_0^{T_{ au}} \mathrm{e}^{-
ho u} du$$
 and $d_{ au}(s)=\int_s^{T_{ au}} \mathrm{e}^{(g^m-
ho)u} du$

- If $\beta = 0$
 - maximize lifetime income: $A_{\tau}(s_{\tau}) = 0$
 - $s_{ au}$ independent of productivity
- If $\beta > 0$, $\bar{c} > 0$
 - Schooling depends upon productivity through c_{τ} , c_{τ}^{m}
 - $U'(c_{\tau}) c_{\tau}$ decreasing
 - $\uparrow c_{\tau}$ holding c_{τ}^m/c_{τ} constant $\rightarrow \uparrow s_{\tau}$

Back

First Order Conditions for ℓ_{τ} and \mathbf{n}_{τ}

• First order conditions for ℓ_{τ}

$$lpha V'\left(\ell_{ au}
ight) = rac{\phi}{1-\gamma}rac{1}{1-n_{ au}-\ell_{ au}}U'\left(c_{ au}
ight)c_{ au}\left(rac{c_{ au}^{m}}{c_{ au}}
ight)^{\sigma}$$

• First order conditions for n_{τ}

$$(1-\phi)\left(z_{ au}^{n}
ight)^{\sigma}\left(n_{ au}
ight)^{\sigma-1}\left(1-n_{ au}-\ell_{ au}
ight)=rac{\phi}{1-\gamma}(c_{ au}^{m})^{\sigma}$$

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- Fit model to U.S. time series of
 - Schooling
 - Workweek
- Average 2 percent growth in income per worker
- Nonmarket hours data

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Calibration

- Schooling data from Goldin and Katz (2008)
 - Years of school completed at age 35 for 1876- to 1975-generation
- Market hours data from Kendrick (1961), McGrattan and Rogerson (2004) and Whaples (1990)
 - Hours worked per worker
- Nonmarket hours from Aguiar and Hurst (2007)
 - Transform hours per person into hours per worker
 - 25.6 hours per worker in 1965 and 22.5 in 2005

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Calibration

- $\rho = 0.04$
- $\psi = 0.3$, $\gamma = 0.1$ (Bils and Klenow, 2000)
- $\sigma = 0.4$ (McGrattan et al, 1997)

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- $\rho = 0.04$
- $\psi = 0.3$, $\gamma = 0.1$ (Bils and Klenow, 2000)
- $\sigma = 0.4$ (McGrattan et al, 1997)
- $z_{\tau}^{m} = e^{g^{m}(\tau 1795)}$
- $z_{\tau}^{n} = \sum_{i=0}^{3} a_{i} (z_{\tau}^{m})^{i}$

Back 20/44

- $\rho = 0.04$
- $\psi = 0.3$, $\gamma = 0.1$ (Bils and Klenow, 2000)
- $\sigma = 0.4$ (McGrattan et al, 1997)
- $z_{\tau}^{m} = e^{g^{m}(\tau 1795)}$
- $z_{\tau}^{n} = \sum_{i=0}^{3} a_{i}(z_{\tau}^{m})^{i}$
- $T_{\tau} = a_T + b_T \tau$ Estimate a_T and b_T using U.S. data on years at school + years at work (Hazan, 2009 and Goldin and Katz, 2008)

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Remaining parameters

$$\omega = (\bar{c}, \phi, \mu, \theta, \alpha, \beta, g^m, a_0, a_1, a_2, a_3)'$$

$$\min_{\omega} \sum_{ au=1880}^{1915} \left(rac{s_{ au}}{s_{ au}^{ extit{data}}} - 1
ight)^2 + \sum_{ au=1795}^{1965} \left(rac{1 - \ell_{ au} - n_{ au}}{1 - \ell_{ au}^{ extit{data}} - n_{ au}^{ extit{data}}} - 1
ight)^2 + M'(\omega) M(\omega),$$

where

$$M(\omega) = \begin{pmatrix} \frac{y_{1965}/y_{1795}}{e^{0.02 \times 170}} - 1\\ n_{1968}/22.5 - 1\\ n_{1938}/25.6 - 1 \end{pmatrix}$$

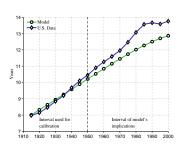
▶ See detail

▶ See parameters

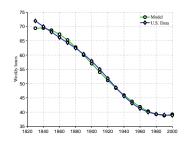
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Calibration - Results



Years of schooling completed at age 35



Weekly Hours

➤ See time allocation

▶ See value of schooling

See value of 7

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Experiments

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Baseline Experiment

- Compute 10 economies that differ by
 - Level and growth rate of market productivity z_{1795}^m and g^m
 - Life expectancy in 1950 and 2005
- Discipline
 - Estimate cross-sectional relation between GDP & Life Expectancy in 1950 and 2005
 - 2. Using step 1, find z_{1795}^m and g^m to match dist. of GDP/cap. in 1950 and 2005

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Baseline Results for 1950

	Dec	Rel Inc	Life Exp	Leis Hrs	Home Hrs	Mkt Hrs	Schl Yrs	ē∕c	ĉ/c	Accounting
										Cross-Section
	1	0.05	20.9	1.3	45.2	65.5	2.2	0.77	1.007	0.90
	2	0.07	24.1	4.8	39.1	68.0	2.6	0.66	1.008	0.87
	3	0.09	26.4	2.3	41.2	68.5	3.7	0.72	1.004	0.92
1	4	0.12	29.2	9.4	33.9	68.7	3.7	0.56	1.006	0.79
9	5	0.17	32.5	10.0	31.9	70.1	5.0	0.54	1.004	0.67
5	6	0.21	34.5	11.4	30.2	70.4	5.8	0.50	1.004	0.71
0	7	0.24	35.7	13.5	28.8	69.7	6.1	0.47	1.004	0.66
	8	0.38	40.1	18.5	25.2	68.3	8.0	0.37	1.003	0.50
	9	0.58	44.1	30.0	22.8	59.2	9.0	0.21	1.003	0.35
	10	0.81	47.3	41.8	23.1	47.1	9.5	0.09	1.003	0.30
	U.S.	1.00	49.3	44.9	23.3	43.8	10.2	0.07	1.003	-

 Model accounts for 90% of difference with U.S. schooling for 1st decile

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Baseline Results for 2005

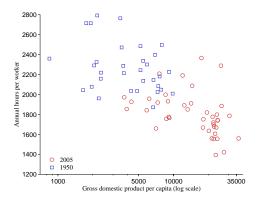
	Dec	Rel Inc	Life Exp	Leis Hrs	Home Hrs	Mkt Hrs	Schl Yrs	ō∕c	ĉ/c	Accounting
										Time Series
	1	0.06	35.6	15.2	29.4	67.4	5.2	0.45	1.004	0.64
	2	0.05	38.5	16.6	29.7	65.6	5.0	0.44	1.005	0.42
	3	0.21	42.9	27.5	22.2	62.3	9.6	0.23	1.003	0.97
2	4	0.10	44.1	27.4	25.1	59.5	7.0	0.28	1.004	0.47
0	5	0.22	48.2	33.9	22.3	55.8	10.1	0.16	1.003	0.53
0	6	0.31	50.3	36.4	21.6	54.0	11.7	0.12	1.003	0.76
5	7	0.34	51.0	38.0	21.6	52.4	12.0	0.10	1.003	0.75
	8	0.61	54.0	41.7	21.3	49.0	14.4	0.05	1.002	0.89
	9	0.71	56.0	46.1	22.4	43.5	14.1	0.03	1.002	0.80
	10	0.77	57.4	49.4	23.4	39.2	13.0	0.02	1.002	0.91
	U.S.	1.00	58.7	48.9	23.1	40.0	13.0	0.02	1.002	-

- Model accounts for 64% of observed growth rate of s in 1st decile
- Elasticity of schooling to gdp: 0.52 (v. 0.76 in data) \rightarrow 0.35 (v. 0.28)

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Work Hours Across Countries

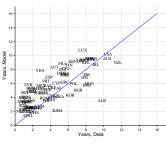


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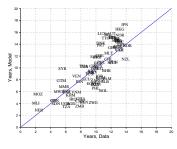
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Schooling-Country by Country Implications

Years of Schooling, Model v. Data



1950.
$$R^2 = 78\%$$



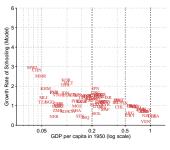
2005.
$$R^2 = 81\%$$

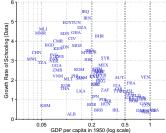
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Schooling-Country by Country Implications

Growth Rate of Schooling by 1950 Income





Model Data

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Experiments

- Experiment 1: The effect of equal productivity growth
- Experiment 2: The effect of equal productivity growth AND equal change in life expectancy

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${\it Experiments}$

			E	kperimen	t 1	Experiment 2				
	Dec	Rel Inc	Mkt Hrs	Schl Yrs	Accounting	Rel Inc	Mkt Hrs	Schl Yrs	Accounting	
					Cross Section				Cross Section	
	1	0.05	65.4	2.2	0.90					
	2	0.07	67.9	2.8	0.85					
	3	0.09	69.0	3.3	0.99					
1	4	0.12	69.4	3.9	0.78					
9	5	0.17	68.2	4.6	0.72					
5	6	0.21	66.6	5.2	0.80					
0	7	0.24	65.1	5.5	0.76					
	8	0.38	58.5	6.9	0.76					
	9	0.58	51.4	8.3	0.55					
	10	0.81	46.4	9.5	0.33					
	U.S.	1.00	43.8	10.2	-					
					Time Series				Time Series	
	1	0.06	67.4	5.3	0.65					
	2	0.08	65.0	6.1	0.51					
	3	0.10	62.8	7.1	0.80					
2	4	0.12	59.5	7.6	0.51					
0	5	0.16	55.1	8.9	0.48					
0	6	0.19	52.3	9.6	0.66					
5	7	0.21	50.4	9.9	0.65					
	8	0.33	44.4	11.3	0.75					
	9	0.51	40.2	12.3	0.70					
	10	0.75	38.9	12.8	0.89					
	U.S.	1.00	40.0	13.0	=					

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${\it Experiments}$

			E	kperimen	t 1	Experiment 2			
	Dec	Rel Inc	Mkt Hrs	Schl Yrs	Accounting	Rel Inc	Mkt Hrs	Schl Yrs	Accounting
					Cross Section				Cross Section
	1	0.05	65.4	2.2	0.90	0.05	65.4	2.2	0.90
	2	0.07	67.9	2.8	0.85	0.07	67.9	2.8	0.85
	3	0.09	69.0	3.3	0.99	0.09	69.0	3.3	0.99
1	4	0.12	69.4	3.9	0.78	0.12	69.4	3.9	0.78
9	5	0.17	68.2	4.6	0.72	0.17	68.2	4.6	0.72
5	6	0.21	66.6	5.2	0.80	0.21	66.6	5.2	0.80
0	7	0.24	65.1	5.5	0.76	0.24	65.1	5.5	0.76
	8	0.38	58.5	6.9	0.76	0.38	58.5	6.9	0.76
	9	0.58	51.4	8.3	0.55	0.58	51.4	8.3	0.55
	10	0.81	46.4	9.5	0.33	0.81	46.4	9.5	0.33
	U.S.	1.00	43.8	10.2	-	1.00	43.8	10.2	-
					Time Series				Time Series
	1	0.06	67.4	5.3	0.65	0.06	66.9	4.4	0.51
	2	0.08	65.0	6.1	0.51	0.08	64.5	5.1	0.40
	3	0.10	62.8	7.1	0.80	0.09	62.0	5.7	0.58
2	4	0.12	59.5	7.6	0.51	0.11	58.8	6.5	0.39
0	5	0.16	55.1	8.9	0.48	0.15	54.3	7.6	0.36
0	6	0.19	52.3	9.6	0.66	0.18	51.4	8.2	0.50
5	7	0.21	50.4	9.9	0.65	0.20	49.6	8.7	0.50
	8	0.33	44.4	11.3	0.75	0.31	43.7	10.2	0.61
	9	0.51	40.2	12.3	0.70	0.49	39.8	11.7	0.61
	10	0.75	38.9	12.8	0.89	0.74	38.8	12.6	0.85
	U.S.	1.00	40.0	13.0	-	1.00	40.0	12.9	-

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Experiments

- Experiment 1: The effect of equal productivity growth
 - Cross Section: accounts for 90% of diff. with U.S. schooling in 1950 (v. 90 in baseline)
 - Time Series: accounts for 65% of growth rate (v. 64% in baseline)
 - Productivity growth differences across countries are small
 - Effect of labor margin on returns to schooling

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Experiments

- Experiment 1: The effect of equal productivity growth
 - Cross Section: accounts for 90% of diff. with U.S. schooling in 1950 (v. 90 in baseline)
 - Time Series: accounts for 65% of growth rate (v. 64% in baseline)
 - Productivity growth differences across countries are small
 - Effect of labor margin on returns to schooling
- Experiment 2: The effect of equal productivity growth AND equal change in life expectancy
 - Cross Section: same as Exp. 1 by construction
 - Time series: accounts for 51% of growth rate (v. 64% in baseline)
 - Elasticity: 0.51 in 1950 \rightarrow 0.34 in 2005

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Conclusion

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Conclusion

- We developed a model of human capital accumulation to assess the quantitative importance of productivity and life expectancy in explaining differences in educational attainment across countries and over time
- The model accounts for 90 percent of the difference in schooling between rich and poor countries in 1950 and 64 percent of the increase in schooling levels over time in poor countries
- The model generates a faster increase in schooling levels in poor than in rich countries, explaining the convergence in cross-country schooling levels observed in the data

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Conclusion

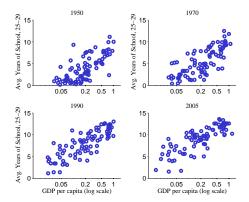
- Model implies faster increase in schooling associated with stronger decline in hours of work, hence, even though human capital increases, per capita income may or may not increase
- This suggest empirical relationship between schooling and per capita income growth across countries, as pioneered for example by Benhabib and Spiegel (1994), does not provide an accurate assessment of the importance of human capital for development
- Our results imply that improving education and welfare in poor countries hinges more on solving their productivity gap with rich countries than pursuing often emphasized educational policies aimed at solving institutional and other frictions

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Extra Material

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Schooling Across Countries and Over Time



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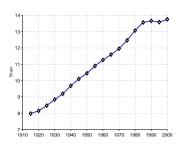
Deciles

- 1. MWI,CHN,MMR,TZA,MLI,KHM,BGD,NER
- 2. PAK, IND, UGA, VNM, ZMB, CMR, KEN, ZWE
- 3. THA, SDN, KOR, IDN, EGY, MLT, ALB, CIV
- 4. GHA, PHL, TUN, MOZ, ROM, SEN, LKA, DZA, JAM
- 5. MYS, IRQ, IRN, JOR, BRA, BGR, TUR, CYP, ECU
- 6. GRC, JPN, BOL, CRI, GTM, PRT, BRB, BHR, COL
- 7. HKG,SAU,ESP,PER,MEX,POL,HUN,ZAF,SYR
- 8. ISR,IRL,ITA,TTO,AUT,CHL,FIN,ARG
- 9. URY, ISL, FRA, BEL, NOR, NLD, SWE, DNK
- 10. GBR.AUS.CAN.VEN.NZL.LUX.CHE.USA

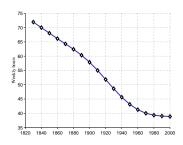
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Calibration

• HP-filtered schooling and hours



Years of schooling completed at age 35



Weekly hours

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Calibration

Income

$$y_{\tau} = z_{\tau}^{m} e^{35 \times g^{m}} (1 - n_{\tau} - \ell_{\tau}) H(s_{\tau}, x_{\tau})$$

- Model's output is decision by generation
- Associate s_{τ} with actual schooling of generation τ
- Associate n_{τ} with actual hours at date $\tau + 35$
- Compute 171 generations from 1795 to 1965
- Match to hours from 1830 to 2000
- Match to schooling of generation 1880 to 1915

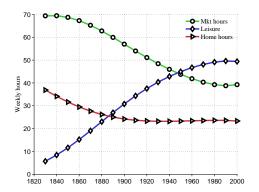
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Calibration

```
Preferences \rho=0.04,\ \bar{c}=0.03,\ \phi=0.10 \sigma=0.40,\ \alpha=0.68,\ \beta=0.71,\ \mu=0.23 Technology z_{1795}^m=1.0,\ g^m=0.019 a_0=0.03410,\ a_1=0.00097,\ a_2=0.00463,\ a_3=-0.00010 \gamma=0.1,\ \psi=0.30,\ \theta=0.06
```

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Time Allocation in Baseline Model



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Consumption Value of Schooling

• We compute $\hat{c}_{ au}$ such that

$$\int_{0}^{T_{\tau}} e^{-\rho t} \left[U(c_{\tau}) + \alpha V(\ell_{\tau}) \right] dt + \beta W(s_{\tau}) =$$

$$\int_{0}^{T_{\tau}} e^{-\rho t} \left[U(\hat{c}_{\tau}) + \alpha V(\ell_{\tau}) \right] dt + \beta W(s_{\tau} - 1)$$

• Find $\hat{c}_{\tau}/c_{\tau} < 1.01$

To compensate for one less year of school, less than a 1% increase in consumption is required

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Value of Subsistence

- How large is \bar{c} ?
- Subsistence can be obtained from income (market) or home production
- We compute \bar{c}/c_{τ}
- Decline from 63% (1800) to 2% (2000)
- Consistent with expenditure share of food: 5.2% in 1996

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