The Baby Boomers and the Productivity Slowdown*

Guillaume Vandenbroucke†

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Abstract

The entry of baby boomers into the labor force in the late 1960s and 1970s slowed growth for physical and human capital per worker because young workers have little of both. Thus, the baby boom could have contributed to the productivity slowdown. I build and calibrate a model à la Huggett et al. (2011) with exogenous population growth, life expectancy, retirement and TFP to evaluate this theory. The baby boom alone accounts for 53% of the slowdown in the period 1964-69, 18% in 1970-74 and 6% in 1975-79.

Keywords: Demography, baby boom, aggregate productivity, productivity slowdown, human capital.

JEL classification: E24, J11, J24

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†Research Division, Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63166, USA. Email: guillaumevdb@gmail.com.

1 Introduction

This paper discusses the effects of demography on aggregate productivity in the United States. By demography I mean, primarily, the age composition of the population, and by aggregate productivity I mean labor productivity, that is gross domestic product (GDP) per worker. The events motivating this discussion are the baby boom on the one hand and the productivity slowdown of the 1960s and 1970s on the other hand—see Figure 1. The term productivity slowdown refers to the *decline* of the growth rate of productivity that started in the 1960s and ended in the late 1970s. The question at the center of this paper is: How and how much, if at all, did the baby boom contribute to the productivity slowdown? In other words, did the baby boom lead to a *declining* rate of productivity growth in the 1960s and 1970s? And, if so, by how much?

During the 1960s and 1970s the average worker became younger because of the baby boom. Since young workers have less human capital and fewer assets than older worker, one expects that human and physical capital per worker slowed down during the 1960s and 1970s. This (age) composition effect could have resulted in a productivity slowdown, even if total factor productivity (TFP) growth remained constant. To the best of my knowledge this mechanism has not been emphasized in the existing literature. Was this effect indeed present? How large was it? Did general equilibrium effects magnify or dampen the composition effect?

Growth accounting cannot answer these questions for two reasons. First, there are no direct measures of human capital or TFP. Second, even if a direct measure indicated a slowdown in the decades following the baby boom, growth accounting would be silent as to what part of it, if any, should be ascribed to the baby boom.

To fix ideas suppose that output is produced via a Cobb-Douglas aggregate production function such as: $Y = K^\alpha(zNh)^{1-\alpha}$, where Y is GDP, K represents total physical capital, N is the number of workers and h represents human capital per worker, respectively. The term z is a labor-augmenting parameter. Since the distinction between labor-augmenting technology and TFP is irrelevant in the case of the Cobb-Douglas technology, I will also refer to z as TFP. Output per worker is

$$y = k^\alpha(zh)^{1-\alpha}, \tag{1}$$

where $y \equiv Y/N$ and $k \equiv K/N$. Although y and k are directly, albeit imperfectly, measured, neither z nor h are observable. Hence the difficulty in assessing the role of the baby boom, which theoretically acts to slow down k and h, versus the role of z.[1] Figure 2 shows that,

[1] Absent a theory of TFP, I consider TFP and demography to be two independent forces.

indeed, the growth rate of k declined during the 1970s. It also reveals that zh, computed as a residual given data on y and k, slowed down. But again, it is not possible to disentangle the contributions of z and h from this figure.

In Section 2 I conduct an empirical exercise which indicates that the baby boom caused a slowdown in human capital per worker during the 1960s and 1970s. This result is "model-free" in the sense that it does not appeal to any notion of economic theory such as optimization or equilibrium. It is not "assumption-free," however. It obtains under the assumption, among others, that the rental rate of human capital is invariant to demographic changes.

To circumvent the need for these assumptions I introduce a model in Section 3. The model resembles that of Huggett et al. (2011) and features the following components: endogenous human capital accumulation à la Ben-Porath (1967), endogenous physical capital accumulation, a neo-classical production function, an overlapping generation structure with exogenous retirement and, finally, a bequest motive whereby parents make a transfer to their offspring during the last period of their lives. The motivation for endogenous human and physical capital accumulation is to allow potential equilibrium effects to play a role. The motivation for bequests is to ensure that the model does not exaggerate the effect of the baby boom by imposing that human capital wealth is the only source of wealth. Finally, prices are determined by marginal products. So, there is no need to assume that the rental rate of human capital is invariant to demographic changes.

I calibrate the model's balanced-growth path to U.S. data in Section 4. Along the balanced growth path the age composition, life expectancy and retirement age are constant, as well as the growth rate of TFP. In Section 5 I compare four transition paths. Each one features changes in one driving variable at a time. In the first, the age composition of workers changes as in the U.S. data. This experiment replicates the effect of the baby boom on U.S. workers. In the second, life expectancy changes as in the U.S. data. In the third, it is the retirement age that changes as in the U.S. data. Finally, for the fourth transition path, I proceed in two steps. First, I find a sequence of growth rates of TFP such that the model's growth rate of GDP per worker equals that of the U.S. economy along a transition where, simultaneously, the age composition, life expectancy and retirement age change as in the U.S. data. Second, I compute a transition path by letting TFP grow as measured, absent any changes in the other driving variables.

The baby boom alone implies a sizeable decline in the growth rate of productivity. Namely, 53.2% of the actual decline in 1965-69 (relative to 1960-64) is accounted for by the model with the baby boom as the only driving variable. During the 1970-74 period the baby boom, on its own, accounts for 17.8% of the actual decline. Finally, during the 1975-1979 period, the baby boom accounts for 6.1% of the actual decline. These three periods encompass the entirety of the productivity slowdown. Four mechanisms interact to explain the effect of the baby boom:

1. Composition effect on human capital per worker — The increasing proportion of young workers with low human capital implies that human capital per worker slows down.

2. Composition effect on physical capital per worker — The increasing proportion of young workers with fewer assets than the average worker implies that the stock of physical capital per worker slows down.

3. Equilibrium effect on human capital per worker — The slowdown of both human and physical capital per worker imply a slowdown of the rental rate of human capital and an increase of the rate of interest. This, in turn, implies that the value of human capital is lower for baby boomers than it was for previous generations. As a result, baby boomers accumulate less human capital, conditional on age, than previous generations. This contributes to slowing down human capital per worker.

4. Equilibrium effect on physical capital per worker — The higher rate of interest implies that, conditional on age, each worker holds more assets than on the balanced-growth path. This dampens the slowdown of physical capital per worker.

I conduct a decomposition exercise revealing that the equilibrium effect on human capital accounts for 30% of the total effect of the baby boom on human capital per worker. I also find that, even though the equilibrium effect on physical capital dampens the productivity slowdown, it is not enough to offset the composition effect.

I find that life expectancy increases productivity growth while the retirement age has an ambiguous effect. As life expectancy alone increases, the length of retirement increases too. This induces individuals to save more and, thus, physical capital grows faster than along the balanced-growth path. As the retirement age alone increases there are two opposing forces at work, however. On the one hand the increasing retirement age raises the returns to human capital accumulation because individuals can rip the benefits of human capital for a longer period. On the other hand, the length of retirement decreases and, thus, individuals save less than along the balanced growth path. Quantitatively, TFP is the second driver behind of productivity slowdown in 1965-69 (accounting for 42.6% on its own) and the main driver in 1970-74 (92.2%) and in 1975-76 (93.9%).

Finally, I conduct an additional experiment combining the baby boom, the increasing life expectancy and the rising age of retirement experiments. I find that the combination of these demographic trends accounts for 55.3% of the slowdown in 1965-69, 6.7% in 1970-74 and 3% in 1975-79. The remainder is explained by TFP.

The main lesson from these findings is that the baby boom contributed noticeably to the productivity slowdown in its early phase but less so in its later phases. This results emphasizes that heterogeneity matters for the measurement of productivity. One could have attributed the productivity slowdown (especially in the early 1960s) entirely to a "fundamental" productivity issue, i.e. a decline of the growth rate of TFP while, in fact, it resulted in part from the changing age composition of the workforce.

Relation to existing literature

The literature on the productivity slowdown is vast. The interested reader can find surveys of this literature in, for example, Cullison (1989) or the 1988 special issue of the *Journal of Economic Perspective*. I sketch below a subset of the arguments made over many decades.

The price of energy, namely oil, which soared after 1973, has often been cited as a major contributor to the slow productivity of the 1970s—see, for instance, Jorgenson (1988), Griliches (1988), Jorgenson and Fraumeni (1983) and Nordhaus (2004). A different explanation is proposed by Greenwood and Yorukoglu (1997), namely the rise of the information age: New technologies are not operated efficiently at first, implying a slowdown in productivity following their introduction. Hornstein and Krusell (1996) also propose a theory of learning that generates a productivity slowdown. They, in addition, emphasize measurement problems that could explain the slowdown. Baily and Gordon (1988) is an influential paper bringing forth measurement issues in the analysis of the productivity slowdown. More recently, Duernecker et al. (2019) emphasize the importance of structural change in explaining past slowdown of U.S. productivity: the reallocation of production to sectors with low productivity growth, particularly services. Bloom et al. (2020) argue that ideas are becoming harder to find and that this is a force toward slower economic growth.

More directly related to this paper, some research focused on the quality of the aggregate labor input during the productivity slowdown. Ho and Jorgenson (1999, Tables 4 and 6), for instance, note that labor quality slowed down noticeably during the years of the productivity slowdown. Darby (1984) and Aaronson and Sullivan (2001) make similar arguments. This part of the literature, therefore, points to a slowdown in input factors (as opposed to a slowdown in TFP), as contributing to the productivity slowdown. Another related paper by Jaimovich and Siu (2009) documents the correlation between the age composition of the labor force and business cycle volatility. They extend their analysis to "the medium term business cycle," into which category the 1960s-1970s slowdown falls. At this frequency they find that the effect of demography on volatility is small.

Feyrer (2011) also proposes a model where the baby boom caused the productivity slowdown. He relies on a Lucas (1978) span-of-control model where the entry of baby boomers implies a fall in management quality and, therefore, a slowdown in productivity. The theory I present shares some resemblance with that of Feyrer (2011) but is silent about management per se. It is more general, however, as it explains a slowdown in labor quality not only for managers but for the average worker. It is also consistent with earnings dispersion, an issue that Feyrer does not discuss but that is central to the literature on human capital accumulation. Finally it emphasizes the link between the baby boom and the slowdown in physical capital that Feyrer (2011) does not emphasize.[2]

From an empirical perspective, Feyrer (2007, 2008) provides evidence of the effect of the age structure of the working population and productivity. Exploiting data from a panel of 87 countries between 1960 and 1990, he finds that large cohorts of workers aged 15-39 are associated with periods of lower productivity. The results are significant and stable across various estimation methods. This empirical approach uses the fact that the age composition of a population is exgenous to current economic conditions when one abstracts from migration issues. Thus, Feyrer's work provide model-free evidence of the connection between demography and productivity growth, and points out that the productivity slowdown was not just a U.S. affair. Bloom et al. (2001) and Young (2005) are also examples of paper emphasizing the importance of demography for economic growth in countries other than the U.S.

2 Empirical analysis

In this section I present a "model-free" calculation to evaluate whether aggregate human capital per worker slowed down, as a result of the baby boom, during the productivity slowdown. The value of this exercise is two-fold. First, it leads to the conclusion that, indeed, aggregate human capital per worker slowed down as a result of the baby boom. Thus, it sheds some light on Figure 2: The slowdown of the zh term was for a part due to a slowdown in human capital per worker. Second, it spells out the assumptions under which this conclusion is warranted and, therefore, motivates the model developed in Section 3 as a way to avoid the limitations of these assumptions.

I use the World Klems dataset which contains the number of employed workers, the average

[2] The rate of return for physical capital is exogenous in Feyrer's model.

hourly compensation, and the average weekly hours worked for each sex, eight age groups and six levels of education in 31 industries from 1947 to 2010. I restrict the analysis to the following five age groups: 18-24, 25-34, 35-44, 45-54, 55-64. The variable $a \in \{1, 2, 3, 4, 5\}$, represents an age group. I also consider the following six educational attainment levels: (1) 8th grade or less, (2) grades 9-12 no diploma, (3) high school graduate, (4) some college no degree or an associate degree, (5) bachelor of art or science, and (6) more than a bachelor's degree. I define the variable $e \in \{1, 2, 3, 4, 5, 6\}$ to represent educational attainment. Finally, the variable s represents a worker's sex (1 for men and 0 for women). I aggregate the data across industries.

I use $E_t(a, s, e)$ to denote the real compensation of a worker of age a, sex s and education e. The function $P_t(a, s, e)$ represents the distribution of employed workers at date t, thus $\sum_{\{a,s,e\}} P_t(a, s, e) = 1$. I define

$$Q_t(a) = \sum_{\{s,e\}} P_t(a, s, e),$$

the marginal distribution of age at date t, and

$$R_t(s, e|a) = P_t(a, s, e)/Q_t(a),$$

the joint distribution of sex and education, conditional on age at date t. Using these distributions, I define

$$X_t = \sum_{\{a,s,e\}} \ln\left(E_t(a, s, e)\right) P_t(a, s, e)$$

and

$$\hat{X}_t = \sum_{\{a,s,e\}} \ln\left(E_t(a, s, e)\right) Q_{1947}(a) R_t(s, e|a),$$

so that X_t represents average (log) earnings at date t and $\hat{X}_t$ is a counterfactual measure of average (log) earnings, under the assumption that the marginal distribution of age remains that of 1947. Note that, by construction, $X_{1947} = \hat{X}_{1947}$.

Let $H_t(a, s, e)$ describe a mapping from a worker's characteristics to his human capital, and let w_t denote the rental rate for human capital. I make the following assumptions:

1. A worker's earnings are of the form $E_t(a, s, e) = w_t H_t(a, s, e)$,
2. w_t is invariant to changes in Q_t,
3. H_t is invariant to changes in Q_t,

Assumption 1 stipulates that a worker's earnings are the product of human capital and its rental rate, and that there is only one rental rate.[3] Assumptions 2 and 3 stipulate that the rental rate of human capital and the mapping from characteristics to human capital remain unchanged when the age distribution is changing.

Under Assumptions 1, 2 and 3 the difference $X - \hat{X}_t$ is

$$X_t - \hat{X}_t = \sum_{\{a,s,e\}} \ln(H_t(a,s,e))P_t(a,s,e) - \sum_{\{a,s,e\}} \ln(H_t(a,s,e))Q_{1947}(a)R_t(s,e|a). \tag{2}$$

That is, $X_t - \hat{X}_t$ is the difference between aggregate (log) human capital per worker and the aggregate (log) human capital per worker that would have prevailed if the marginal distribution of age had remained constant at 1947.

Figure 3 plots $X_t - \hat{X}_t$ for three categories of workers. It transpires that human capital per worker falls below its hypothetical (no-baby boom) path during the 1960s and 1970s. Thus, under assumptions 1, 2 and 3, one can conclude that the baby boom caused a noticeable slowdown in aggregate human capital. The slowdown is more pronounced for college-educated workers because they have steeper earnings profiles: All else equal, an increase in the proportion of young workers should reduce human capital per worker more for college-educated workers than for the others. Quantitatively, aggregate human capital per worker fell 2% below its hypothetical path in 1980. As per assumptions 2 and 3, however, this result overlooks potential equilibrium effects. In the remainder of the paper I develop and analyze an equilibrium model to remedy this.

3 Model

3.1 Demography

Time is discrete and indexed by t. There is no uncertainty. The economy is populated by overlapping generations of individuals. Age-1 individuals at date t (generation t) live for J_t periods. They work from age 1 to $R_t - 1$ and retire at age R_t. The retirement age is exogenous.

In each generation individuals differ in their endowment of human capital, h_1, and their ability to further accumulate human capital, x. An individual's ability does not change with age. A type is a pair: $s \in \mathcal{S} \equiv \{(h_1, x) : h_1 > 0,\ x > 0\}$, and $S(s)$ denotes the time-invariant distribution

[3] It is standard to assume that earnings are of form $w \times H$. See, for instance, Heckman et al. (1998) or Bowlus and Robinson (2012). The assumption of a unique rental rate implies that the human capital of workers of different types are perfect substitute. This is not critical since the analysis can be carried out independently for different types of workers.

of types.

Let $p_{t,j}(s)$ denote the population of type s and age j from generation t. Each age-1 member of generation t gives births to $(1+\gamma_p)^M$ children who become age-1 individuals in period $t+M$. I assume that type-s individuals give births to type-s children. The laws of motion for $p_{t,j}(s)$ are

$$p_{t+M,1}(s) = (1+\gamma_p)^M p_{t,1}(s) \quad (3)$$
$$p_{t+M,j}(s) = p_{t+M,1}(s). \quad (4)$$

I will refer to γ_p as the rate of population growth, even though this is a slight abuse of language.[4]

3.2 Production

Output is produced via a constant-returns-to-scale technology:

$$Y_t = K_t^\alpha (z_t H_t)^{1-\alpha}$$

with $\alpha \in (0,1)$ and where K_t represents physical capital, H_t represents human capital and z_t represents labor-augmenting technological progress. Physical capital depreciates at rate δ_K each period.

3.3 Individuals

The preferences of age-1 individuals from generation t are represented by

$$U_t(s) = \sum_{j=1}^{J_t} \beta^{j-1} \frac{c_{t,j}(s)^{1-\sigma}}{1-\sigma} + \beta^{J_t-1}\theta \frac{b_t(s)^{1-\sigma}}{1-\sigma}. \quad (5)$$

In this expression $c_{t,j}(s)$ indicates consumption at age j and $b_t(s)$ indicates bequest per child. Bequests are made during the last period of life. The parameter β is the subjective discount factor and θ measures the strength of the bequest motive. I use "warm-glow preferences" (where parents derive utility directly from the bequest, instead of caring about their children's utility) as the bequest motive for simplicity.

Let $Q_{t+\tau}$ denote the date-t value of a good available in period $t+\tau$. It is defined as $Q_t = 1$ and $Q_{t+1} = Q_t/(1+r_{t+1})$ where r_{t+1} is the interest rate running from period t to $t+1$. Members

[4] The rate of population growth is γ_p when J_t is constant.

of generation t receive a bequest from their parents (generation $t-M$) at age $J_{t-M}-M$, thus the lifetime budget constraint is

$$\sum_{j=1}^{J_t} Q_{t+j-1}c_{t,j}(s) + Q_{t+J_t-1}(1+\gamma_p)^M b_t(s) = W_{t,1}(h_1,x) + Q_{t+J_{t-M}-M-1}b_{t-M}(s), \tag{6}$$

where $W_{t,1}(h_1,x)$ indicates human capital wealth at age 1 for a type $s=(h_1,x)$. Human capital wealth is the solution of the following wealth maximization problem:

$$Q_{t+j-1}W_{t,j}(h,x) = \max_{n\in(0,1]} Q_{t+j-1}w_{t+j-1}h(1-n) + Q_{t+j}W_{t,j+1}(h',x) \tag{7}$$

$$\text{s.t.} \quad h' = (1-\delta_H)h + x(nh)^{\phi} \tag{8}$$

$$W_{t,R_t}(h,x) = 0 \tag{9}$$

where w_{t+j-1} is the rental rate of human capital. Note a few points. First, the optimization problem (7)-(9) defines human capital wealth independently of consumption-saving decisions. This is because individuals do not value leisure, and because credit markets are perfect. Second, Equation (7) defines human capital wealth in period $t+j-1$ as the sum of the (discounted) labor income of period $t+j-1$ and the (discounted) human capital wealth of period $t+j$. Note the term $(1-n)$ in the description of labor income: As individuals allocate more time to human capital accumulation labor income decreases. The trade-off in choosing n is therefore between lower current income and future higher income. Third, Equation (8) describes the law of motion for human capital. The depreciation rate is δ_H, and $x(nh)^{\phi}$ is the production function for new human capital. Note the role of ability: the higher the ability x, the higher the marginal return to time spent in human capital accumulation. Fourth, Equation (9) is a terminal condition. It stipulates that, upon retiring, an individual's human capital looses its value.

Let $h_{t,j}(s)$ and $n_{t,j}(s)$ denote optimal human capital and the time spent accumulating it. Finally, let $a_{t,j}(s)$ denote net asset positions that derives from the period budget constraints:

$$c_{t,j}(s) + a_{t,j+1}(s) + (1+\gamma_p)^M b_t(s)\mathbb{I}\{j=J_t\} \\ = w_{t+j-1}\mathbb{I}\{j<R_t\} + (1+r_{t+j-1})a_{t,j}(s) + b_{t-M}(s)\mathbb{I}\{j=J_{t-M}-M\}.$$

3.4 Equilibrium

3.4.1 *Optimization*

The maximization of utility (5) subject to the lifetime budget constraint (6) yields the standard Euler equation

$$\left(\frac{c_{t,j}(s)}{c_{t,j+1}(s)}\right)^{-\sigma} = \beta\frac{Q_{t+j-1}}{Q_{t+j}}.$$

The first-order condition for bequests is

$$\beta^{J_t-1}\theta b_t(s)^{-\sigma} = c_{t,1}^{-\sigma}(1+\gamma_p)^M Q_{t+J_t-1}$$

where the left-hand side is the marginal utility benefit of the bequest and the right-hand side is the marginal cost. I show in Appendix A that the wealth maximization problem (7)-(9) admits an interior solution of the form

$$W_{t,j}(h,x) = A_{t,j}(x) + B_{t,j}h,$$

where

$$\begin{aligned} B_{t,R-1} &= w_{t+R_t-2}, \\ B_{t,j} &= w_{t+j-1} + \frac{Q_{t+j}}{Q_{t+j-1}}B_{t,j+1}\left(1-\delta_H\right) \text{ for } j < R_t - 1. \end{aligned} \tag{10}$$

The term $B_{t,j}$ is the marginal value of human capital: it measures the effect of a unit of human capital on human capital wealth. Note that $B_{t,j}$ is independent of a worker's type. Human capital is worth the same for each worker because the human capital of any worker is a perfect substitute for the human capital of any other worker. A worker's type matters only for the ease with which human capital can be accumulated. The first-order condition for an interior solution for n is

$$w_{t+j-1} = \frac{Q_{t+j}}{Q_{t+j-1}}B_{t,j+1}\phi x\left(nh\right)^{\phi-1}. \tag{11}$$

The left-hand side of this equation represents the marginal cost of allocating time to human capital accumulation: the forgone wage. The right-hand side represents the marginal benefit: The change in the stock of human capital multiplied by the marginal value of human capital, discounted to the current period.[5]

Finally, profit maximization and competitive factor markets imply that the interest rate and

[5] There exists the possibility of a corner solution where $n = 1$. In this case, the value function is $Q_{t+j-1}W_{t,j}(h,x) = Q_{t+j}W_{t,j+1}\left((1-\delta_H)h + F^H(h,x), x\right)$.

the rental rate of human capital are determined, respectively, by

$$r_t = \alpha \left(\frac{K_t}{z_t H_t} \right)^{\alpha-1} - \delta_K \tag{12}$$

$$w_t = z_t(1-\alpha) \left(\frac{K_t}{z_t H_t} \right)^{\alpha} \tag{13}$$

where δ_K is the rate of depreciation of physical capital.

3.4.2 *Definition of equilibrium*

A competitive equilibrium is a sequence of prices, $\{w_t, r_t\}_t$, allocations for firms, $\{K_t, H_t\}_t$ and allocations for households of all age, cohorts and types, $\{c_{t,j}(s), h_{t,j}(s), n_{t,j}(s), a_{t,j}(s), b_t(s)\}_{t,j,s}$, such that firms and households optimize given prices and such that markets clear.

The market clearing condition for labor at date t is

$$\sum_{\{j:j<R_{t-j+1}\}} \sum_{s\in\mathcal{S}} p_{t-j+1,j}(s) h_{t-j+1,j}(s) \left(1 - n_{t-j+1,j}(s)\right) = H_t. \tag{14}$$

The left-hand side of Equation (14) represents the total supply of labor services, that is, the sum of human capital-hours across all working cohorts at date t and across all types. The right-hand side is the demand for labor services. The resource constraint is

$$\sum_{\{j:j\leq J_{t-j+1}\}} \sum_{s\in\mathcal{S}} p_{t-j+1,j}(s) c_{t-j+1,j}(s) + K_{t+1} = Y_t + (1-\delta_K)K_t. \tag{15}$$

Finally, the market clearing condition on the savings market is

$$\sum_{\{j:j\leq J_{t-j+1}\}} \sum_{s\in\mathcal{S}} p_{t-j+1,j}(s) a_{t-j+1,j+1}(s) = K_{t+1}. \tag{16}$$

3.4.3 *Balanced growth*

Suppose the labor-augmenting technology grows at a constant rate:

$$z_{t+1} = (1+\gamma_z) z_t.$$

Suppose, in addition, that life expectancy is constant, $J_t = J$, that the age of retirement is constant $R_t = R$, and that γ_p is constant. The competitive equilibrium exhibits, then, the balanced-growth path property. That is, output, Y_t, physical capital, K_t, and the supply of labor services, $z_t H_t$, grow at rate $(1+\gamma_z)(1+\gamma_p)-1$. The interest rate, r_t, is constant and the rental rate for human capital, w_t, grows at rate γ_z. Individual consumption, asset holdings and bequests, $c_{t,j}(s)$, $a_{t,j}(s)$ and $b_t(s)$, grow at rate γ_z.[6] The derivation of the balanced-growth path equations is in Appendix B.

3.5 Discussion

On within-cohort heterogeneity

I introduced two layers of heterogeneity, i.e. h_1 and x, beside age in the model, why? Suppose that a change in prices—for instance induced by the baby boom—incites workers to accumulate human capital differently. Data on earnings by age, particularly the positive correlation between age and earnings inequality, indicates that workers of the same age but with different "types" will adjust their human capital differently. Can the (average) quantitative magnitude of these effects be disciplined in a model where agents are homogeneous within cohorts? I adopted the view that it was better to model within-cohort heterogeneity, and relied on the work of Huggett et al. (2006, 2011) to do so. These authors demonstrated that a two-dimensional type-heterogeneity within cohorts, i.e., initial human capital and ability to learn, is critical to matching earnings statistics by age, and in particular earnings inequality. Modeling and calibrating this model along the same lines as Huggett et al. (2011) implies that the response of the average worker in the model, in the face of changing prices, is disciplined.

On schooling

I did not introduce schooling in the model, while the rise in years of schooling that took place in the U.S. throughout the 20th century is large and well documented. To see how this matters suppose each cohort starts working with more human capital than the previous one, thanks to a rise in schooling. Could the slowdown of human capital per worker be mitigated because the average worker of the baby boom generation, in spite of being younger, would also be more educated?

If schooling shifted the distribution of initial human capital to the right for each new cohort, the rise in schooling would contribute to the growth rate of productivity in a way that is similar to the contribution of TFP, i.e., each cohort would be more productive than the preceding one because it would have higher human capital. For schooling to offset the effect of the baby

[6] That is $c_{t+1,j}(s) = (1+\gamma_z)c_{t,j}(s)$, $a_{t+1,j}(s) = (1+\gamma_z)a_{t,j}(s)$ and $b_{t+1}(s) = (1+\gamma_z)b_t(s)$.

boom, it would therefore be necessary that schooling *accelerated* for the baby boom generation and not just increased as for any other generation. Figure 4 shows, however, that the opposite is true: The baby boom generation marks a noticeable slowdown in the growth of educational attainment in the United States. Two remarks follow. First, the calculations presented in the remainder of the paper *understate* the effect of the baby boom. Modeling schooling in a way that fits the U.S. data would imply a stronger slowdown of human capital per worker than implied by the current model without schooling. The effect of the baby boom on productivity would, therefore, be stronger. Second, the slowdown in years of schooling shown in Figure 4 is, interestingly, consistent with the model presented here since the model implies a slowdown of the value of human capital for age 1 workers, $B_{t,1}$, following the baby boom (see Section 5 and Figure 14). In a model where schooling decisions would depend on the value of human capital at the start of one's worklife, the slowdown of $B_{t,1}$ caused by the baby boom would imply a slowdown in years of school. Kong et al. (2018) is an example of such a model.

On hours

In section 1 I defined productivity as GDP per worker and, in the quantitative part of this paper, I compare output per worker from the model to GDP per worker in the data. Figure 5 shows that GDP per hour in the U.S. data exhibits a slowdown similar, albeit less pronounced, than that of GDP per worker. The motivation for focusing on GDP per worker stems from the Ben-Porath model being silent about the labor-leisure choice and the difficulty, therefore, in comparing the model's output to GDP per hour.[7] Since the slowdown in GDP per worker is more pronouced than the slowdown in GDP per hour in the U.S. data, focusing on the former presents more of a challenge for the model.

4 Calibration

I calibrate the initial balanced-growth path of the model to U.S. data. A model period is a year. I set the growth rate of population γ_p to reflect the growth rate of the U.S. population during the twentieth century: $\gamma_p = 0.01$. I set $\gamma_z = 0.02$ to imply a 2% growth rate of output per worker along the balanced growth path. I assume that age 1 in the model corresponds to age 20 in the U.S. data. Life expectancy at age 20 in the 1930s is about 48, so I set $J = 48$.[8]

[7] That the Ben-Porath model is silent about the labor-leisure choice has been pointed out before. See, for instance, Huggett et al. (2006) and Manuelli and Seshadri (2014).

[8] Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at www.mortality.org or www.humanmortality.de. The life expectancy at age 20 is the remaining number of years one can expect to live, conditional on being alive at age 20.

Lee (2001, Table 1) indicates that the expected length of retirement for males reaching age 20 in 1930 was about 9 years. Thus, I set $R = 39$ for the initial balanced growth path.

I use standard values for the curvature of the utility index and the capital share: $\sigma = 2.0$ and $\alpha = 0.36$. I choose the rate of depreciation of physical capital to imply a 7% investment-to-capital ratio in the balanced-growth path.[9] This implies $\delta_K = 0.04$. I set the curvature in the human capital technology to $\phi = 0.7$, which is in the middle of the range of estimates for this parameter (see Browning et al., 1999). Following Huggett et al. (2006), I set the rate of depreciation for human capital, δ_H, to 1% per year.

The remaining parameters are the subjective discount factor β, the weigth of bequests in utility, θ, and parameters pertaining to the joint distribution of ability and initial human capital, S. I represent S via a discretization of a bi-variate log-normal distribution: $LN(E_S, \Sigma_S)$. The discretization uses a 20×20 grid for initial human capital and ability. I choose $\omega \equiv (\beta, \theta, E_S, \Sigma_S)$ to minimize a distance between some model-generated moments and their empirical counterparts. The moments are:

1. The average labor earnings by age, denoted $M_j^1(\omega)$;
   Note that $M_j^1(\omega)$ is a $(R-1) \times 1$ vector;

2. The Gini coefficient of labor earnings by age, denoted $M_j^2(\omega)$;
   $M_j^2(\omega)$ is a $(R-1) \times 1$ vector;

3. The variance of the logarithm of labor earnings by age, denoted $M_j^3(\omega)$;
   $M^3(j)$ is a $(R-1) \times 1$ vector;

4. The capital-to-output ratio, denoted $M^4(\omega)$;
   $M^4(\omega)$ is a scalar.

5. The intergenerational transfer share of net worth: denoted $M^5(\omega)$;
   $M^5(\omega)$ is a scalar.

Data on average labor earnings, the Gini coefficient and the variance of log-earnings by age are provided by Huggett et al. (2011). The target figure for the capital-to-output ratio is 3. The target figure for the intergenerational transfer share of net worth is 30%—see Lee and Seshadri

[9]Data from the Bureau of Economic Analysis reveal that the investment-to-capital ratio in the 1950s (the earliest period available) was in the neighborhood of 7%. See BEA, Table 5.10. "Changes in Net Stock of Produced Assets (Fixed Assets and Inventories)."

(2019). The calibration procedure involves solving the following minimization problem:

$$\min_{\omega} \sum_{i=1,2,3} \sum_{j=1}^{R-1} \left(M_j^i(\omega)/\mathbf{M}_j^i - 1\right)^2 + \left(M^4(\omega)/\mathbf{M}^4 - 1\right)^2 + \left(M^5(\omega)/\mathbf{M}^5 - 1\right)^2,$$

where bold-face symbols denote data. Table 1 shows the resulting parameters. The parameters imply a correlation of 0.91 between ability and initial human capital. The coefficient of variation for the marginal distribution of initial human capital is 0.59; for the marginal distribution of ability it is 0.38. Figure 6 shows the model's fit to the U.S. data. Note in Figure 6 that both assets and earnings increase with age, in a proportion that replicates the U.S. data well. The increase in assets (north-west quadrant of Figure 6) implies that, holding the age distribution of assets fixed, there will be less physical capital per worker in an economy with a larger proportion of young workers. The increase in earnings (north-east quadrant of Figure 6) indicates that young workers have less human capital than old workers. Thus, holding the age distribution of human capital fixed, there will be less human capital per worker in an economy with a larger proportion of young workers.

5 Analysis

I compute four transition paths. Along each path the economy is subjected to changes in one driving variables at a time, instead of remaining along its balanced-growth path. There are four driving variables: (i) the rate of population growth, γ_p; (ii) life expectancy, J_t; (iii) the retirement age, R_t; and (iv) TFP growth, γ_z. I describe below how these driving variables are disciplined.

Changes in population growth — I choose a series of values for γ_p to replicate the proportion of young workers in the U.S. data. Figure 7 shows the implied proportion of young workers in the model and its empirical counterpart. I assume that in the long-run γ_p returns to its initial value of 1%.

Changes in life expectancy — I use life expectancy at age 20 from the Human Mortality Database. I assume that life expectancy continues to increase linearly, in the long-run, at the same rate as in the data, without exceeding 63.[10] Figure 8 shows life expectancy.

Changes in the retirement age — Lee (2001) indicates that the expected length of retirement was 9 years for the generation that was 20 in 1930, and 16 for the generation that was 20 in 1990. I assume that the expected length of retirement continues to increase linearly, in the

[10] The Census Bureau estimates life expectancy at age 20 in 2050 to be 63.20.

long-run, at the same rate as between 1930 and 1990, but that it does not exceed 25 years.[11] I obtain the retirement age by subtracting the expected length of retirement from life expectancy. Figure 8 shows the retirement age.

Changes in TFP growth — I compute the growth rate of TFP as a residual. Specifically, I find a sequence for γ_z such that the model's growth rate of GDP per worker equals that of the U.S. over the period 1955-1989 (by 5-year intervals) along a transition where, simultaneously, the age composition, life expectancy and retirement age change as in the U.S. data. I assume that γ_z returns to its initial value of 2% in the long-run. Figure 9 shows the growth rate of TFP measured in this way.[12]

5.1 The effect of the baby boom

I analyze, first, the transition path where only the population growth rate changes. I label this the "baby boom-only" experiment. Figure 10 shows the paths of the rental rate of human capital, w_t, and the rate of interest, r_t, relative to their balanced-growth path. The interest rate increases to 5.7% above its balanced-growth path while the rental rate of human capital falls 2% below its balanced growth path. This last observation indicates that, along this transition, w_t grows at a lower rate than along the balanced-growth path, i.e. it slows down.

Figure 11 shows output per worker, human capital per worker and physical capital per worker divided, respectively, by their balanced-growth path values. All three quantities slow down when the proportion of young workers increases. Output per worker (productivity) falls 5.7% below its balanced-growth path, human capital falls 3.7% and physical capital falls 9.1%. The slowdown of factors accounts for all the productivity slowdown since the growth rate of TFP is constant by assumption.

What are the mechanisms contributing to the slowdown of factors? I start with human capital per worker and construct

$$X_{H,t} = \sum_{j=1}^{R-1} \sum_{s \in \mathcal{S}} \frac{p_{t-j+1,j}(s)}{N_t} h_{t-j+1,j}(s) \left(1 - n_{t-j+1,j}(s)\right), \tag{17}$$

where $N_t = \sum_{j=1}^{R-1} \sum_{s \in \mathcal{S}} p_{t-j+1,j}(s)$ is the number of workers at date t. Thus, $X_{H,t}$ is human capital per worker. I include the term $1 - n$ to reflect the fact that the input used in production

[11]According to the Social Security Administration a male born on 01/01/2020 will spend between 21 and 25 years in retirement, and a woman between 23 and 27.

[12]This implies that the model's rate of growth or productivity exactly equals that of the U.S. economy during for the periods 1955-59, 1960-64, 1965-69, 1970-74, 1975-79, 1980-84, and 1985-89.

is the product of human capital and time. I also construct

$$\hat{X}_{H,t} = \sum_{j=1}^{R-1} \sum_{s \in \mathcal{S}} \frac{p_{1933,j}(s)}{N_{1933}} h_{t-j+1,j}(s) \left(1 - n_{t-j+1,j}(s)\right), \tag{18}$$

which represents what human capital per worker would have been if the age distribution of workers had remained fixed at its initial (1933) value. Figure 12 shows $X_{H,t}$ and $\hat{X}_{H,t}$ relative to human capital per worker along the balanced-growth path. A departure of $X_{H,t}$ from the balanced-growth path indicates the total effect of the baby boom on human capital per worker. A departure of $\hat{X}_{H,t}$ from the balanced-growth path indicates only the effects operating through individual decisions. I label this the "equilibrium effect" on Figure 12. The difference between the total effect and the equilibrium effect is a composition effect due to the changing age distribution of the population.[13]

Figure 12 reveals that the slowdown of human capital per worker during the 1960s and 1970s results mostly from the changing composition of the workforce. Changes in patterns of human capital accumulation play a secondary role, as can be deduced from the smaller changes in the Equilibrium-effect line in comparison to the Total-effect line. The Equilibrium effect is not negligible, however. At the trough, when human capital per worker is 3.7% below its balanced growth path, the Equilibrium-effect line is 1.1% below. Thus, the Equilibrium effect accounts for 30% of the slowdown of human capital per worker; the remainder is a composition effect.

The slowdown due to the Equilibrium effect is caused by price movements described in Figure 10. Recall that the value of human capital at any age is given by $B_{t,j}$ (see Equation 10), which is the present value (adjusted for depreciation) of the rental rate of human capital over a worker's remaining work life. Both the slowdown of the rental rate of human capital and the acceleration of the rate of interest work to reduce the value of human capital relative to the balanced-growth path. Hence the reduced human capital accumulation. The Composition effect results from young workers having less human capital than old workers.

I now turn to physical capital per worker and define

$$X_{K,t} = \sum_{j=1}^{R-1} \sum_{s \in \mathcal{S}} \frac{p_{t-j+1,j}(s)}{N_t} a_{t-j+1,j}(s) \tag{19}$$

and

$$\hat{X}_{K,t} = \sum_{j=1}^{R-1} \sum_{s \in \mathcal{S}} \frac{p_{1933,j}(s)}{N_{1933}} a_{t-j+1,j}(s), \tag{20}$$

[13] The dashed blue line in Figure 12, that is $X_{H,t}$, is the same as in Figure 11: the total effect of the baby boom on human capital per worker.

which have, for physical capital, the same interpretations as $X_{H,t}$ and $\hat{X}_{H,t}$ have for human capital. Figure 13 shows $X_{K,t}$ and $\hat{X}_{K,t}$ relative to their balanced-growth path equivalent. The figure reveals that the slowdown of physical capital per worker results entirely from the changing age composition of the workforce. The Equilibrium effect of the baby boom would have implied an increase in capital per worker because, facing a higher rate of interest, savings per individual are higher—see Figure 10.

A few points are worth emphasizing at this stage. First, and as noted earlier, the price movements in Figure 10 imply that the value of human capital $B_{t,j}$ falls below its balanced growth value for the baby boomers. This is illustrated in Figure 14 for workers at the start of their lives: $B_{t,1}$. This effect of the baby boom has implications for the study of educational attainment: Suppose that initial human capital, which is exogenous in this model, resulted from a schooling decision. The slowdown of $B_{t,1}$ implies that the value of schooling would slow down for baby boomers because the value of schooling would be determined by the value of human capital at the end of school, i.e., $B_{t,1}$. This implies that (i) the baby boom would, in itself, be a force toward an explanation of the slowdown in schooling noted by Goldin and Katz (2007) and displayed in Figure 4; and (ii) the results presented here *understate* the effect of the baby boom on productivity since, if schooling was modeled, a slowdown in initial human capital will imply a stronger slowdown of human capital per worker.

Second, the slowdown in capital per worker in Figure 11 is stronger than for human capital per worker. This implies that the ratio K/H slows down after the baby boom and, therefore, rationalizes the dynamic of prices in Figure 10. Figure 14 shows the behavior of K/H relative to the balanced-growth path.

Finally, note that the slowdown in human capital per worker implied by the model (3.7%) differs from the slowdown measured via the model free calculation of Section 2 (2.0%). Is this difference entirely due to the Equilibrium effect absent from the calculation of Section 2? To answer this I apply the approach of Section 2 to model-generated data. Figure 15 shows the results. It plots (again) human capital per worker as implied by the model, relative to the balanced-growth path (dashed blue). It also plots (solid green) $X_t - \hat{X}_t$ as defined by Equation (2), but applied to model generated data. At the trough, human capital per worker is 3.7% below its balanced-growth. Relying on earnings data only, one would have computed a 2.7% fall at the same date. Thus, the model free method reveals 73% of the fall. The size of the error is 27%, close to the 30% Equilibrium effect.

5.2 Decomposing the productivity slowdown

I now turn to comparing all four transition paths: (i) The "baby boom-only," where the only driving variable is the changing rate of population growth designed to replicate the share of young workers; (ii) the "life expectancy-only" where the sole driving variables is life expectancy; (iii) the "retirement age-only"where the age of retirement alone changes; and (iv) The "TFP-only," where the only driving variable is the growth rate of TFP.

Table 2 reports the results of these experiments in comparison with the data. The first column, labeled "dy/y," indicates the growth rate of GDP per worker in the U.S. economy during seven 5-year periods from 1955 to 1989. Columns labeled "$\Delta dy/y$," indicate changes in the growth rate of GDP per worker. A negative number in these columns reveals a slowdown of productivity: i.e. a decrease of the growth rate of output per worker. The shaded area of the table corresponds to the three periods of productivity slowdown in the U.S.: 1965-69, 1970-74 and 1975-79. Columns labeled "% of data" indicate the model-to-data ratio of $\Delta dy/y$.

Consider the first period of the productivity slowdown: 1965-69. The growth rate of productivity decreased from 2.66% in the preceding period to 2.19%, a 47 basis point decline. Under the baby-boom only experiment the growth rate of GDP per worker would have declined by 25 basis points, accounting for 53.2% (25/47) of the slowdown during this period. Under the TFP-only experiment, the growth rate of GDP per worker would have declined by 20 basis points, accounting for 42.6% of the slowdown. Under the life expectancy-only experiment, productivity would have accelerated, albeit only slightly. This is because, as life expectancy alone increases, the length of retirement increases as well. This induces individuals to save more and, therefore, the stock of physical capital per worker grows faster than it would otherwise. The retirement age has ambiguous effects on productivity growth. On the one hand, an increase in the age of retirement reduces the length of retirement and, thus, operates in the opposite direction than life expectancy. This is a force toward slower productivity growth. On the other hand, an increase in the age of retirement implies an increase in the value of human capital, $B_{t,j}$ (see Equation 10) because individuals can rip the benefits from human capital over a longer period. This is a force toward an acceleration of human capital per worker and, thus, productivity relative to the balanced growth path. During the first period of the productivity slowdown the former effect dominates the latter.

Turning to the remaining periods of productivity slowdown, it transpires from Table 2 that the baby boom alone accounts for 17.8% of the slowdown during the 1970-74 period, and 6.1% during the 1975-79 period. During these two periods, changes in TFP account for most of the productivity slowdown. Life expectancy acts against the productivity slowdown while the direction of the effect of the retirement age changes.

It is interesting to note that, in the periods following the productivity slowdown, the baby boom-only experiment implies, in line with the U.S. data, some noticeable increase in productivity growth. This is because during this period the proportion of young workers declined (Figure 1) implying the exact opposite of the mechanisms that lead to the slowdown of the preceeding periods. In 1980-84 and then in 1985-89, the growth rate of productivity in the U.S. increased by 29 and 35 basis points, respectively. Under the baby-boom only experiment the increase in productivity growth is 19 and 32 basis points, that is 65.5 and 91.4% of the data. During these periods TFP growth plays a lesser role in explaining changes in productivity growth. Specifically, TFP accounts for 13.8% of the acceleration of productivity in 1980-84, and acts to reduce productivity growth in 1985-89.

5.3 Discussion

Demography versus TFP

I compute one additional transition path where γ_p, life expectancy and the retirement age change as described earlier. Table 3 reports the results and compare them with U.S. data and the TFP-only transition. The message from Table 3 is that demography, on its own, accounts for 55.3% of the slowdown in 1965-69, 6.7% in 1970-74 and 3.0% in 1975-79. In the aftermath of the productivity slowdown, that is in the 1980s, demography accounts for most of the acceleration of productivity: 82.7% in 1980-84 and 111.4% in 1985-89.

There are few to no interactions between TFP and demography. This can be seen from changes in the actual growth rate of productivity (col. 3 of Table 3) that are almost exactly equal to the sum of the slowdowns implied by demography alone on the one hand and TFP alone on the other hand (cols. 5 and 7). I show in Appendix C that the lack of interaction between TFP and demography does not follow from the construction of TFP growth as a residual. Instead, the fact that columns 5 and 7 of Table 3 sum to (almost) exactly column 3 indicates that the mapping from TFP and demography to productivity growth features (almost) no interactions. The same remark can be made about Table 2.

The echo effect of the baby boom

I have discussed how the arrival of baby boomers in the labor force affected productivity. This begs a follow up question: How is their retirement likely to affect productivity?

Figure 16 shows the mean age of workers in both the U.S. data and in transition paths of the model featuring changes in γ_p. The decline before the 1980s (in both data and model) corresponds to the entry of baby boomers into the labor force. The decline after 2020 (in the model) corresponds to their retirement. Note the slowdown in the U.S. data in the late 2010s. Thus, the retirement of baby boomer will have the same effect as their arrival: it is a force toward lower productivity growth. This can be seen in Figure 17. The model predicts that between 2020 and 2040 the baby boom alone would cause U.S. productivity to decline from 7% above its balanced growth path to close to 3% above. This is a decline comparable, in magnitude, to the decline that took place from the mid-1960s to the mid-1980s.

One should keep in mind that Figure 17 does not provide a forecast of productivity growth since it obtains under the assumption of constant TFP growth, constant life expectancy and retirement age. Furthermore, events such as the 2008-2009 recession are not taken into considerations. The main lesson from Figure 17 is thus that the retirement of baby boomers will exert a sizeable downward pressure on productivity growth in the next 20 years.

6 Conclusion

In this paper, I asked: did the baby boom lead to a declining rate of productivity growth in the 1960s and 1970s? And, if so, how and by how much? I built a neo-classical growth model featuring endogenous human capital accumulation à la Ben-Porath (1967), endogenous physical capital accumulation, a neo-classical production function, an overlapping generation structure with exogenous retirement and, finally, a bequest motive. I calibrated the model to U.S. data and computed a transition path with a baby boom replicating that in the U.S. data. I found that, in the absence of changes in the rate of TFP growth, the baby boom alone accounts for 53.2% of the slowdown in 1965-69, 17.8% in 1970-74 and 6.1% in 1975-1979.

The main lesson from this exercise, therefore, is that the baby boom contributed noticeably to the productivity slowdown in its early phase but less so in its later phases, when TFP played the major role.

The exercise I conducted also implied a slowdown in the value of human capital for the baby boomers. Two important remarks follow. First, this means that the contribution of the baby boom to the productivity slowdown is understated by the model. A model with endogenous schooling would imply a stronger decline in human capital per workers. Second, the model suggests a theory of the slowdown in years of schooling documented for the baby boom generation. I leave research on the link between the baby boom and educational attainment in the late 20th century for future work.

Table 1: Parameters

Demography	$J = 48,\ R = 39,\ M = 20,\ \gamma_p = 0.010$
Preferences	$\sigma = 2.00,\ \theta = 0.21,\ \beta = 0.999$
Goods production technology	$\alpha = 0.36,\ \delta_K = 0.04,\ \gamma_z = 0.020$
Human capital technology	$\phi = 0.70$, $\delta_H = 0.01$
Distribution of types	$(E_{h_1}, E_x) = (87.19, 0.25)$
	$(\Sigma_{h_1}, \Sigma_x, \Sigma_{h_1,x}) = (56.58, 0.09, 4.52)$

Table 2: Productivity slowdown: data and models

	U.S.		Model							
			BB		TFP		LE		RA	
	dy/y	$\Delta dy/y$	$\Delta dy/y$	% of data	$\Delta dy/y$	% of data	$\Delta dy/y$	% of data	$\Delta dy/y$	% of data
1955–59	+2.34									
1960–64	+2.66	+0.32	−0.14	−43.8	+0.46	143.8	+0.00	0.0	−0.00	−0.0
1965–69	+2.19	−0.47	−0.25	53.2	−0.20	42.6	+0.01	−2.1	−0.01	2.1
1970–74	+1.30	−0.90	−0.16	17.8	−0.83	92.2	+0.03	−3.3	+0.10	−11.1
1975–79	+0.96	−0.33	−0.02	6.1	−0.31	93.9	+0.06	−18.2	−0.04	12.1
1980–84	+1.25	+0.29	+0.19	65.5	+0.04	13.8	+0.10	34.5	−0.02	−6.9
1985–89	+1.60	+0.35	+0.32	91.4	−0.06	−17.1	+0.04	11.4	+0.05	14.3

Note: BB stands for baby boom-only; TFP stands for TFP-only; LE stands for life expectancy-only; RA stands for retirement age-only; dy/y is the growth rate of GDP per worker. Columns labeled "$\Delta dy/y$" indicate changes in the growth rate of GDP per worker from period to period. Columns labeled "% of data" indicate model-to-data ratios of $\Delta dy/y$. The shaded area indicates the productivity slowdown.
Source: PWT 9.0 and author's calculations.

Table 3: Productivity slowdown: demography versus TFP

	Data		Model			
			BB + LE + RA		TFP	
	dy/y	$\Delta dy/y$	$\Delta dy/y$	% of data	$\Delta dy/y$	% of data
1955–59	+2.34					
1960–64	+2.66	+0.32	−0.15	−46.9	+0.46	143.8
1965–69	+2.19	−0.47	−0.26	55.3	−0.20	42.6
1970–74	+1.30	−0.90	−0.06	6.7	−0.83	92.2
1975–79	+0.96	−0.33	−0.01	3.0	−0.31	93.9
1980–84	+1.25	+0.29	+0.24	82.8	+0.04	13.8
1985–89	+1.60	+0.35	+0.39	111.4	−0.06	−17.1

Note: BB stands for baby boom-only; TFP stands for TFP-only; LE stands for life expectancy-only; RA stands for retirement age-only. dy/y is the growth rate of GDP per worker. Columns labeled "$\Delta dy/y$" indicate changes in the growth rate of GDP per worker from period to period. Columns labeled "% of data" indicate model-to-data ratios of $\Delta dy/y$. The shaded area indicates the productivity slowdown.
Source: PWT 9.0 and author's calculations.

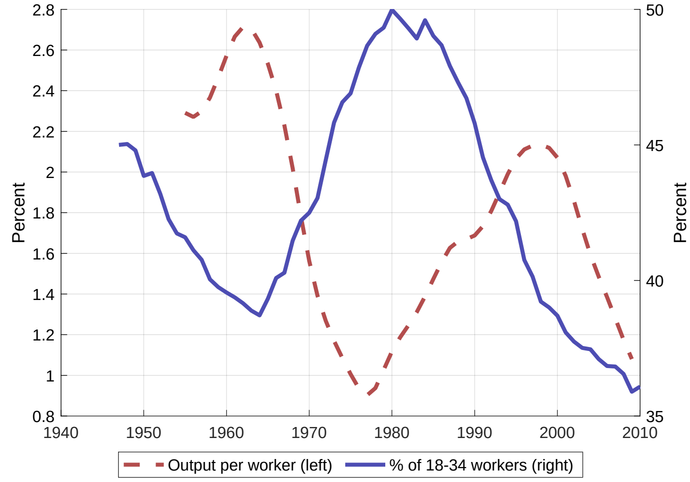

Figure 1: The growth rate of GDP per worker and the proportion of young workers

Note: Moving average of the growth rate of hp-filtered (trend component) real GDP per worker with $\lambda = 6.25$ (Ravn and Uhlig, 2002), and the proportion of 18-34 workers among 18-64 workers.
Source: Penn World Tables 9.0 and World Klems.

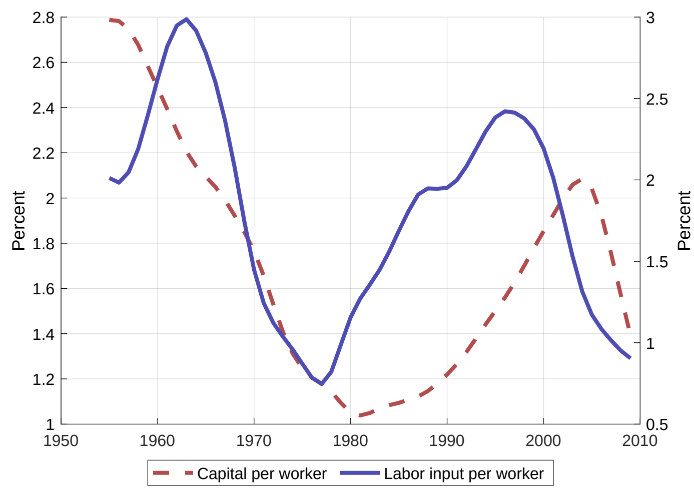

Figure 2: The growth rate of capital and labor per hour

Note: The production function is $y = k^{\alpha}(zh)^{1-\alpha}$; the term zh is computed as a residual given data on y and k. The figure plots moving averages of the growth rate of hp-filtered (trend component) k and zh with $\lambda = 6.25$ (Ravn and Uhlig, 2002).
Source: Penn World Tables 9.0.

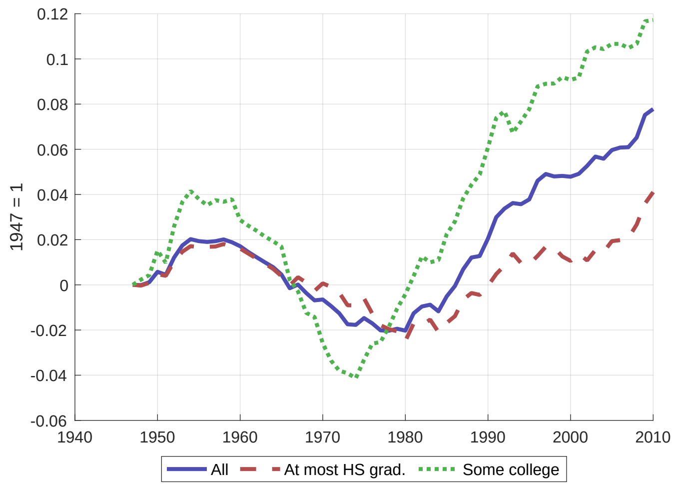

Figure 3: The effect of the age composition of workers by education level

Note: The figure plots $X_t - \hat{X}_t$, as defined in Equation (2), for three categories of workers.
Source: World Klems and author's calculations.

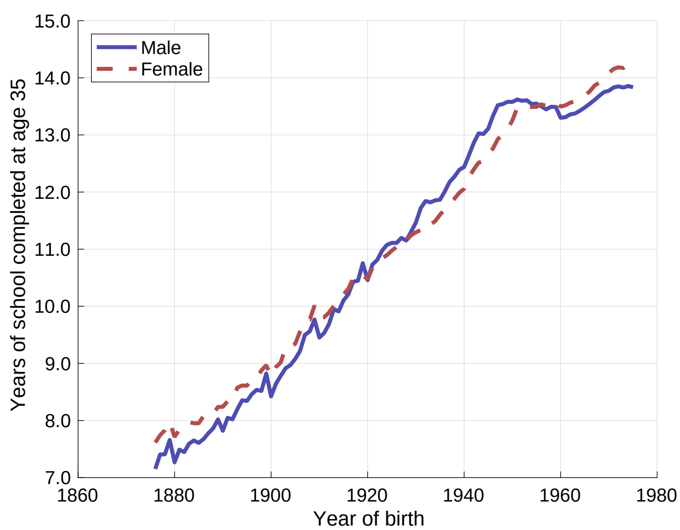

Figure 4: Years of schooling

Note: Years of schooling completed at age 35, by birth cohort.
Source: Goldin and Katz (2007).

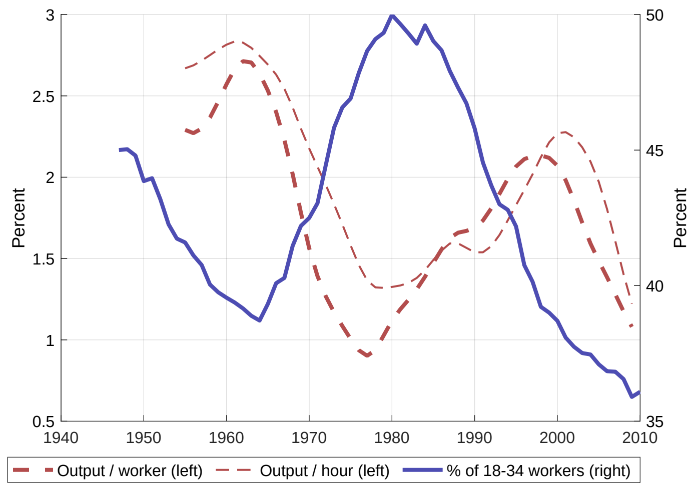

Figure 5: The growth rates of GDP per worker & GDP per hour and the proportion of young workers

Note: Moving average of the growth rate of hp-filtered (trend component) real GDP per worker (and per hour) with $\lambda = 6.25$ (Ravn and Uhlig, 2002), and the proportion of 18-34 workers among 18-64 workers.
Source: Penn World Tables 9.0 and World Klems.

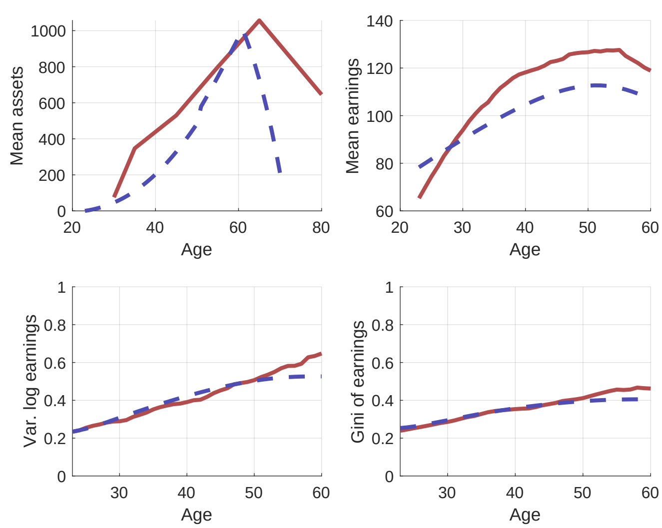

Figure 6: Model fit to U.S. data

Note: Data (solid) and model (dashed). The distribution of assets by age (north-west) is not a target in the calibration, but it is an important moment for the argument in this paper: Younger individuals hold fewer assets than older individuals.
Source: Huggett et al. (2011), Survey of Consumer Finances, and author's calculations.

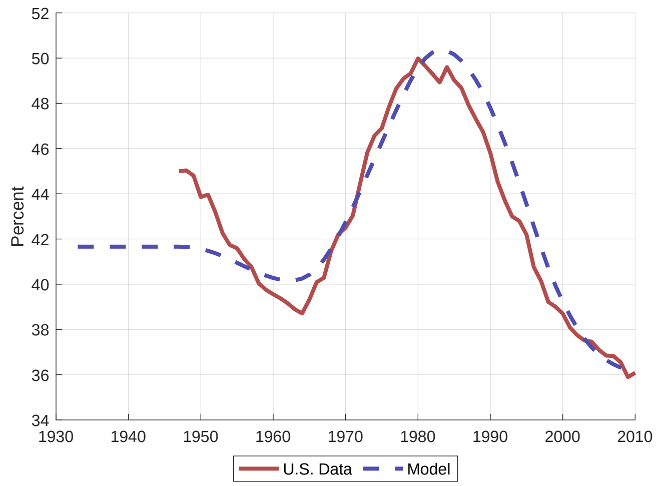

Figure 7: The proportion of young workers

Note: Percent of 18-34 workers in the population of 18-64 workers.
Source: World Klems and author's calculations.

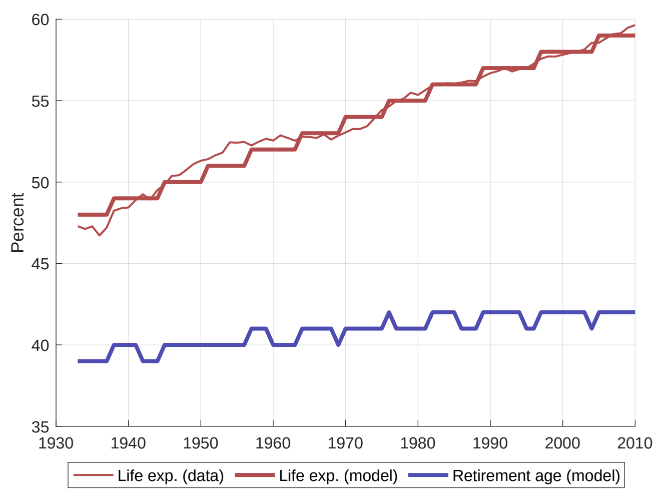

Figure 8: Life expectancy at age 20 and retirement age

Note: Life expectancy at age 20, model v. data, and the retirement age in the model. The retirement age is obtained by subtracting the expected length of retirement from life expectancy.
Source: Human Mortality Database for life expectancy, and Lee (2001, Table 1) for the expected length of retirement. (Human Mortality Database: University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at www.mortality.org or www.humanmortality.de).

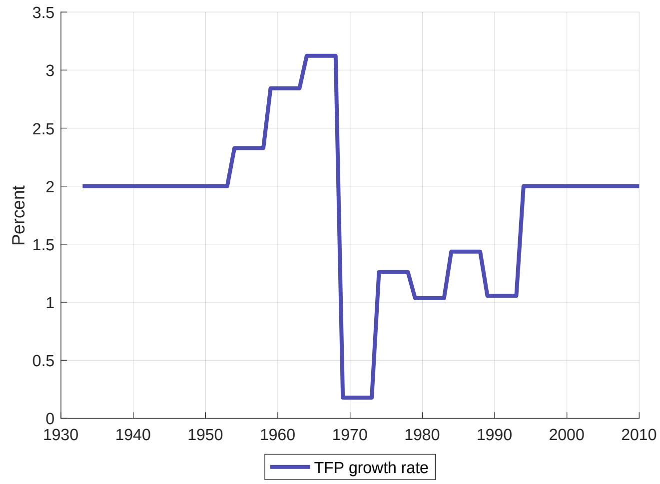

Figure 9: Measured TFP growth

Source: Author's calculations.

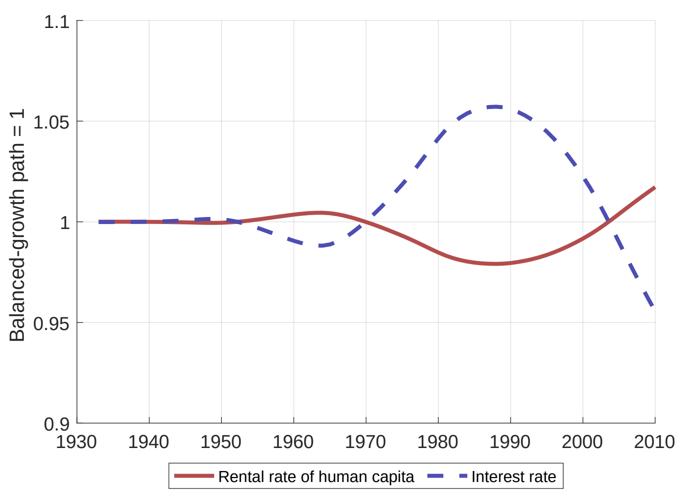

Figure 10: The effect of the baby boom on prices

Note: Baby boom-only / balanced-growth path.
Source: Author's calculations.

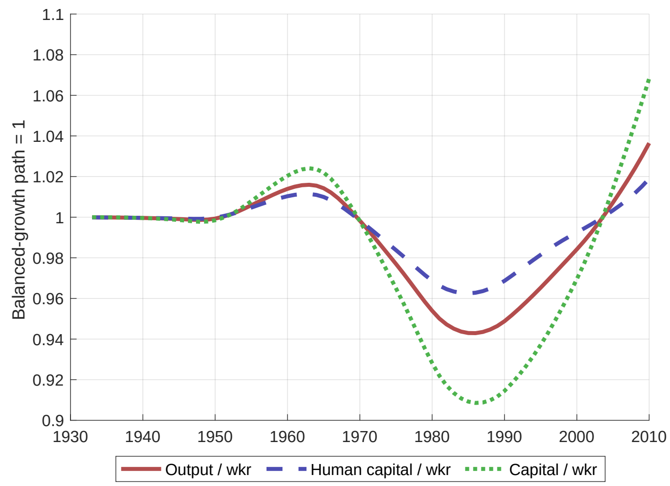

Figure 11: The effects of the baby boom

Note: Baby boom-only / balanced-growth path.
Source: Author's calculations.

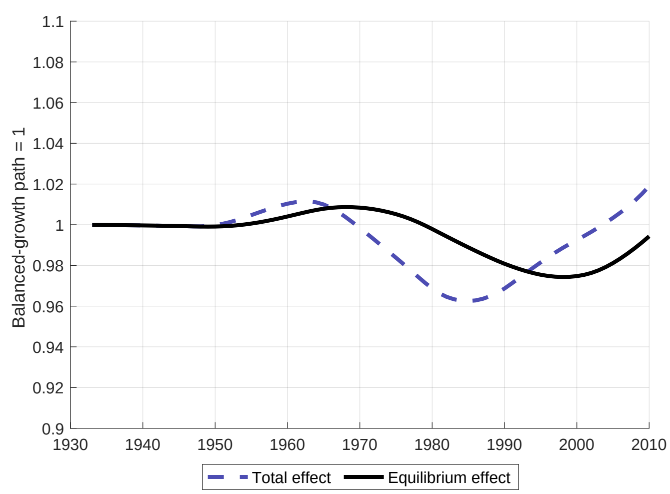

Figure 12: The effect of the baby boom on human capital per worker

Note: The figure shows $X_{H,t}$ (dashed, blue) and $\hat{X}_{H,t}$ (solid, dark) in the baby boom-only experiment, relative to their balanced-growth path equivalent. See Equations (17) and (18) for a definition of these terms.
Source: Author's calculations.

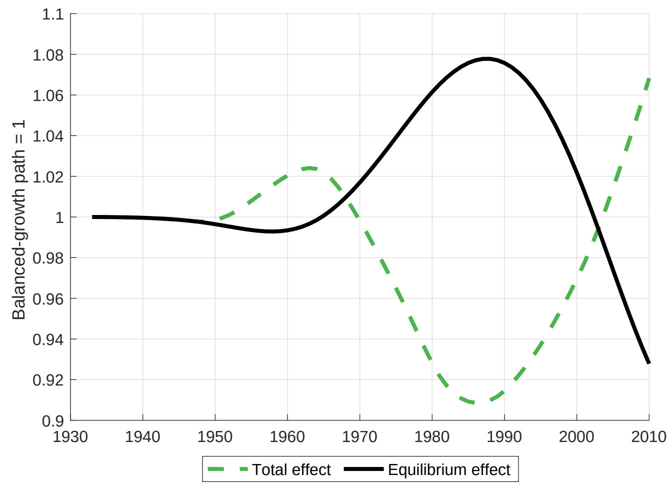

Figure 13: The effect of the baby boom on physical capital per worker

Note: The figure shows $X_{K,t}$ (dotted, green) and $\hat{X}_{K,t}$ (solid, dark) in the baby boom-only experiment, relative to their balanced-growth path equivalent. See Equations (19) and (20) for a definition of these terms.
Source: Author's calculations.

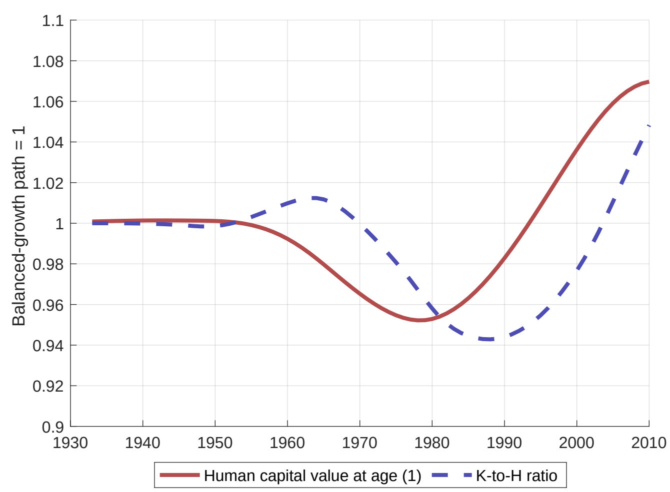

Figure 14: The effect of the baby boom on the value of human capital and the K/H-ratio

Note: Baby boom-only / balanced-growth path.
Source: Author's calculations.

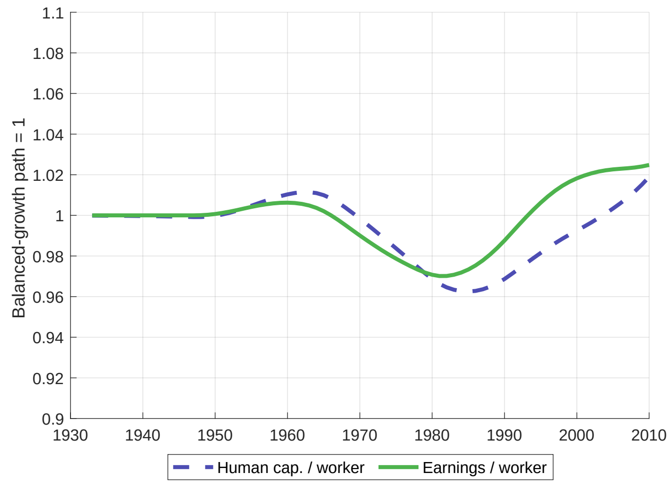

Figure 15: The total effect of the baby boom on human capital and earnings

Note: This calculation is similar to that of Section 2 but uses model-generated data.
Source: Author's calculations.

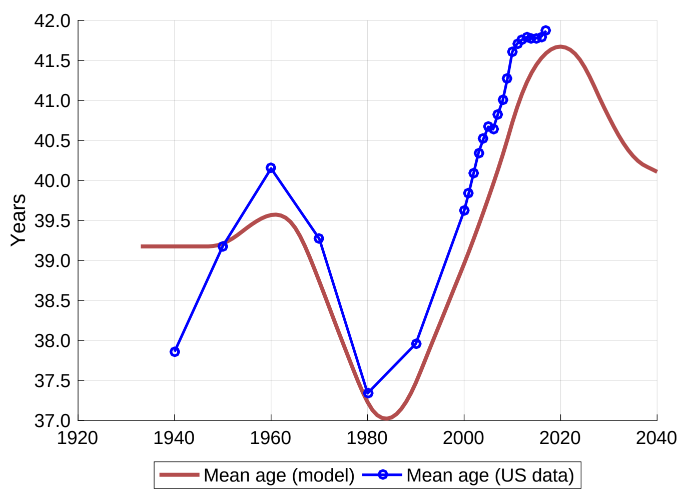

Figure 16: The mean age of workers

Source: IPUMS and author's calculations.

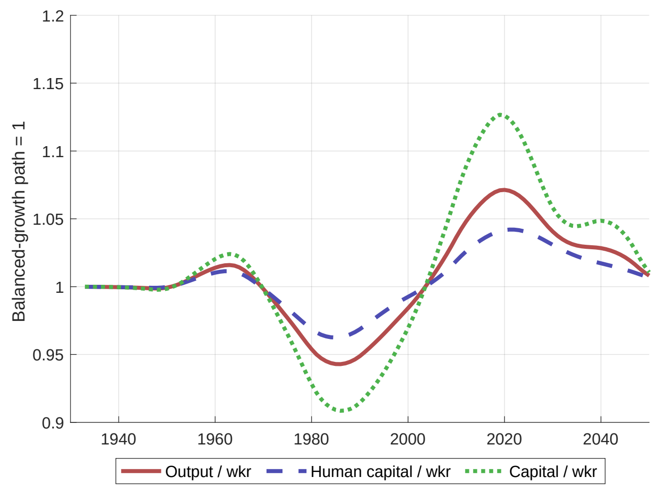

Figure 17: The echo effect of the baby boom

Note: Baby boom-only / balanced-growth path.
Source: Author's calculations.

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A Wealth maximization

Problem (7)-(9) reads

$$\begin{aligned} W_{t,j}(h,x) &= \max_{n\in(0,1]} w_{t+j-1}h(1-n) + \frac{Q_{t+j}}{Q_{t+j-1}} W_{t,j+1}(h',x) \\ \text{s.t.} \quad & h' = (1-\delta_H)h + x(nh)^{\phi} \\ & W_{t,R}(h,x) = 0 \end{aligned}$$

The solution at age $R-1$ is $W_{t,R-1}(h,x) = w_{t+R-2}h$. Write this as $W_{t,R-1}(h,x) = A_{t,R-1}(x) + B_{t,R-1}h$, where $A_{t,R-1}(x) = 0$ and $B_{t,R-1} = w_{t+R-2}$. Suppose now that the solution at age $j+1$ is of the form $W_{t,j+1}(h,x) = A_{t,j+1}(x) + B_{t,j+1}h$; then the optimization problem at age j becomes

$$\begin{aligned} W_{t,j}(h,x) &= \max_{n} w_{t+j-1}h(1-n) \\ & + \frac{Q_{t+j}}{Q_{t+j-1}} \left[A_{t,j+1}(x) + B_{t,j+1}(1-\delta_H)h + B_{t,j+1}x(nh)^{\phi} \right]. \end{aligned}$$

The first-order condition for an interior solution is

$$w_{t+j-1} = \frac{Q_{t+j}}{Q_{t+j-1}} B_{t,j+1}\phi x (nh)^{\phi-1}$$

implying

$$nh = \left(\frac{Q_{t+j}}{Q_{t+j-1}} \frac{B_{t,j+1}}{w_{t+j-1}} \phi x \right)^{1/(1-\phi)}$$

The value function becomes then

$$\begin{aligned} W_{t,j}(h,x) = \max_{n} \; & w_{t+j-1}h - w_{t+j-1} \left(\frac{Q_{t+j}}{Q_{t+j-1}} \frac{B_{t,j+1}}{w_{t+j-1}} \phi x \right)^{1/(1-\phi)} \\ & + \frac{Q_{t+j}}{Q_{t+j-1}} A_{t,j+1}(x) + \frac{Q_{t+j}}{Q_{t+j-1}} B_{t,j+1}(1-\delta_H)h \\ & + \frac{Q_{t+j}}{Q_{t+j-1}} B_{t,j+1} x \left(\frac{Q_{t+j}}{Q_{t+j-1}} \frac{B_{t,j+1}}{w_{t+j-1}} \phi x \right)^{\phi/(1-\phi)}. \end{aligned}$$

which can be written as $W_{t,j}(h,x) = A_{t,j}(x) + B_{t,j}h$ where

$$B_{t,j} = w_{t+j-1} + \frac{Q_{t+j}}{Q_{t+j-1}} B_{t,j+1}(1-\delta_H)$$

and

$$A_{t,j} = \frac{Q_{t+j}}{Q_{t+j-1}} A_{t,j+1}(x) + \left(\frac{1}{\phi} - 1 \right) w_{t+j-1} \left(\frac{Q_{t+j}}{Q_{t+j-1}} \frac{B_{t,j+1}}{w_{t+j-1}} \phi x \right)^{1/(1-\phi)}.$$

B Balanced growth

I characterize the steady state of the economy (balanced-growth path) when γ_z and γ_p are constant, as well as life expectancy J and the retirement age R. Along the balanced-growth path, individual-level variables conditional on age, such as consumption, saving and human capital wealth, grow at rate γ_z. Aggregate-level variables, such as output, the aggregate stock of capital and aggregate consumption, grow at rate $(1+\gamma_p)(1+\gamma_z)-1$. Aggregate labor supply grows at the rate of population growth, γ_p.

B.1 Demography

The laws of motion (3)-(4) are equivalent to

$$\begin{aligned} p_{t,1}(s) &= (1+\gamma_p)\, p_{t-1,1}(s) \\ p_{t,j}(s) &= p_{t,1}(s). \end{aligned}$$

These imply $p_{t,1}(s) = (1+\gamma_p)^{j-1} p_{t-j+1,1}(s)$. It follows that the proportion of age-j individuals of type s in the total population of type s individuals is

$$\pi_j = \frac{(1+\gamma_p)^{1-j}}{\sum_{j'} (1+\gamma_p)^{1-j'}}.$$

Total population is $p_t = \sum_{j=1}^{J} \sum_{s\in\mathcal{S}} p_{t-j+1,j}(s)$. The proportion of age-j and type-s individuals is $p_{t-j+1,j}(s)/p_t = S(s)\,\pi_j$.

B.2 Firms

Suppose that the supply of human capital-hours, that is, $h(1-n)$, is constant given age. That is, $h(1-n)$ varies with age, but workers of the same age in different cohorts supply the same $h(1-n)$. I show below that this is indeed the case along the balanced-growth path. Aggregate labor supply then grows at the rate of population growth, γ_p. Along the balanced-growth path, the output-to-capital ratio, $Y_t/K_t = (z_t H_t/K_t)^{1-\alpha}$, is constant. This implies that the aggregate stock of capital grows at rate $(1+\gamma_z)(1+\gamma_p)-1$. Prices are determined by marginal products:

$$r + \delta_K = \alpha \left(\frac{K_t}{z_t H_t}\right)^{\alpha-1}$$

and

$$w_t = (1-\alpha) z_t \left(\frac{K_t}{z_t H_t}\right)^{\alpha}.$$

Define $\hat{w} = w_t/z_t$, $\hat{K} = K_t/(z_t H_t)$ and $\hat{Y} = Y_t/(z_t H_t)$. Then $\hat{Y} = \hat{K}^{\alpha}$, $r = \alpha \hat{K}^{\alpha-1} - \delta_K$ and $\hat{w} = (1-\alpha)\hat{Y}$.

B.3 Individuals

B.3.1 *The income maximization problem*

When the rate of interest is constant, problem (7)-(9) writes

$$\begin{aligned} W_{t,j}(h,x) &= \max_n \; w_{t+j-1}h\,(1-n) + \frac{1}{1+r}W_{t,j+1}\,(h',x) \\ \text{s.t.} \quad & h' = (1-\delta_H)h + x(nh)^{\phi} \\ & W_{t,R}(h,x) = 0. \end{aligned}$$

Define $\hat{W}_j(h,x) = W_{t,j}(h,x)/z_{t+j-1}$. It follows that

$$\hat{W}_j(h,x) = \max_n \; \hat{w}h\,(1-n) + \frac{1+\gamma_z}{1+r}\hat{W}_{j+1}\,(h',x)\,.$$

and the solution of this problem is $\hat{W}_j\,(h,x) = \hat{A}_j(x) + \hat{B}_j h$, where $\hat{B}_j = B_{t,j}/z_{t+j-1}$ and $\hat{A}_j(x) = A_{t,j}(x)/z_{t+j-1}$. Note that

$$\hat{B}_j = \hat{w} + \frac{(1+\gamma_z)\,(1-\delta_H)}{1+r}\hat{B}_{j+1} = \hat{w}\sum_{\tau=j}^{R-1}\left(\frac{(1+\gamma_z)\,(1-\delta_H)}{1+r}\right)^{\tau-j}. \tag{21}$$

The first-order condition (11) implies then that nh is independent of $\hat{w}$ and depends on age only. It follows that both h and n are independent of $\hat{w}$:

$$h_{t,j}(s) = h_j(s) \;\text{ and }\; n_{t,j}(s) = n_j(s)$$

Thus, labor supply grows at rate γ_p along balanced growth path.

B.3.2 *The consumption-saving problem*

Define $\hat{c}_j(s) = c_{t,j}(s)/z_{t+j-1}$, $\hat{a}_j(s) = a_{t,j}(s)/z_{t+j-1}$ and $\hat{b}(s) = b_t(s)/z_{t+j-1}$. Preferences can then be represented by

$$\hat{U} = \sum_{j=1}^{J}\tilde{\beta}^{j-1}\frac{\hat{c}_j(s)^{1-\sigma}}{1-\sigma} + \tilde{\beta}^{J-1}\theta\frac{\hat{b}(s)^{1-\sigma}}{1-\sigma}$$

where $\tilde{\beta} = \beta\,(1+\gamma_z)^{1-\sigma}$. The lifetime budget constraint (6) becomes

$$\sum_{j=1}^{J}\left(\frac{1+\gamma_z}{1+r}\right)^{j-1}\hat{c}_j(s) + \left(\frac{1+\gamma_z}{1+r}\right)^{J-1}(1+\gamma_p)^M\,\hat{b}(s) = \hat{W}_1(h_1,x) + \left(\frac{1+\gamma_z}{1+r}\right)^{J-M-1}\hat{b}(s).$$

B.3.3 *Equilibrium*

It is convenient to define $\omega_j(s) = p_{t-j+1,j}(s)/H_t$ and to note that it is independent of time. To see this, use Equation (14) and the fact, established above, that h and n are independent of time:

$$\begin{aligned}\omega_j(s) &= \frac{p_{t-j+1,j}(s)/p_t}{\sum_{j'=1}^{R-1}\sum_{s'\in\mathcal{S}} p_{t-j'+1,j'}(s')h_{j'}(s')\left(1-n_{j'}(s')\right)/p_t} \\ &= \frac{\pi_j S(s)}{\sum_{j'=1}^{R-1}\sum_{s'\in\mathcal{S}} h_{j'}(s')\left(1-n_{j'}(s')\right)\pi_{j'}S(s')}.\end{aligned}$$

The labor-market clearing condition, Equation (14), becomes

$$\sum_{j=1}^{R-1}\sum_{s\in\mathcal{S}}\omega_j(s)h_j(s)\left(1-n_j(s)\right) = 1.$$

Dividing the resource constraint, Equation (15), by $z_t H_t$ yields

$$\sum_{j=1}^{J}\sum_{s\in\mathcal{S}}\omega_j(s)\hat{c}_j(s) + \hat{K}(1+\gamma_p)(1+\gamma_z) = \hat{Y} + (1-\delta_K)\hat{K}.$$

Finally, dividing the equilibrium condition on the savings market, Equation (16), by $z_t H_t$ implies

$$\sum_{j=1}^{J}\sum_{s\in\mathcal{S}}\omega_j(s)\hat{a}_{j+1}(s) = \hat{K}(1+\gamma_p).$$

C Decomposition

Does the absence of interaction between the driving forces of the model, noted in Tables 2 and 3, results from the construction of TFP growth as a residual? The answer to this question is no. I show this via a simple counter example. Consider a function $y = M(x,z)$, where $y, x, z \in \mathbb{R}$ and

$$M(x,z) = ax + bz + cxz.$$

where $a, b, c \in \mathbb{R}$. Suppose data on y and x are available at two points in time: (y_0, x_0) and (y_1, x_1). It is immediate that z can be computed as a "residual" via

$$z_i = \frac{y_i - ax_i}{b + cx_i},\ \text{for } i = 0, 1.$$

Such computation mimics the methodology I employ in Section 5, where the growth rate of TFP is measured as a residual given data on the growth rate of productivity, the growth rate of population, life expectancy and the retirement age. Continuing with the logic employed in Section 5, the effect of x is computed as

$$M(x_1, z_0) - M(x_0, z_0) = a(x_1 - x_0) + cz_0(x_1 - x_0),$$

and the effect of z is computed as

$$M(x_0, z_1) - M(x_0, z_0) = b(z_1 - z_0) + cx_0(z_1 - z_0).$$

Let D denote the sum of the two effects: $D = M(x_1, z_0) - M(x_0, z_0) + M(x_0, z_1) - M(x_0, z_0)$. Let Δ denote the total effect of x and z, that is $\Delta = M(x_1, z_1) - M(x_0, z_0)$. It transpires that

$$\Delta - D = c(x_1 - x_0)(z_1 - z_0).$$

The sum of the two effects of x and z, computed separately, equals the total effect whenever $c = 0$ (no interaction between x and z) or x and/or z remain constant. The fact that z is computed as a "residual" does not imply that $z_1 = z_0$ and, thus, is irrelevant. To conclude, the additivity noted from Tables 2 and 3 implies that there are (almost) no interactions between the driving variables of the model in the determination of the growth rate of productivity.