Fertility Shocks and Equilibrium Marriage-Rate Dynamics*

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July 2016

Abstract:

Why did the marriage probability of single females in France after World War 1 rise above its pre-war average, despite a 33% drop in the male/female singles ratio? We conjecture that war-time disruption of the marriage market generated an abnormal abundance of men with relatively high marriage propensities. Our model of matching over the lifecycle, when calibrated to pre-war data and two war-time shocks, succeeds in matching the French time path under the additional assumption of a pro-natalist post-war preference shock. We conclude that endogeneity issues make the sex ratio a potentially unreliable indicator of female marriage prospects.

Keywords: Family Economics, Household Formation, Marriage, Fertility.

JEL Classification: D10, E13, J12, J13, and O11.

*We are grateful for constructive comments from conference participants at REDg Barcelona 2013, LAEF-NYU 2013, Sciences Po Paris 2013, Essex SAM 2013, OLG Days Paris 2014, ENSAI Rennes 2014, Iowa 2014,ZEW Mannheim 2014, Philadelphia Fed 2014, and department seminars at USC, Southampton, Birmingham, SFU, IMF, Laval, UC Louvain, Tel Aviv, Be'er Sheva, Dallas Fed, EIEF(Rome), and EUI(Florence). Knowles thanks the Southampton CPC for encouragement and financial support. The views expressed here are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

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1 Introduction

The idea that female marriage prospects depend critically on the male-to-female ratio among singles (the sex ratio) is central to theories of marriage in both economics and demography. A high sex ratio is expected to generate high female marriage probabilities (denoted "marriage-hazard" rates hereafter), thus improving the prospects of single women. This argument also underlies the use of sex ratios as an empirical tool for the study of household decisions. Exogenous shocks to the sex ratio are rare however, so the hypothesis is not easy to verify. In this light, the dramatic (33%) decline of the sex ratio in France as a result of World War 1 (1914-1918) constitutes a rare opportunity to learn about the aggregate dynamics of marriage rates by examining the response of marriage hazards over time.

Marriage rates per capita were much higher in the decade after the war than before the war, for both men and women. A similar pattern holds for all of the major post-war periods from 1800 to 1945, as Figure 1 makes clear. But marriage rates per capita can increase either because of a higher fraction of singles in the population, or because of higher marriage-hazard rates.[1] The post-war fraction of singles was indeed abnormally high for both sexes after the war, due to a severe war-time decline in marriages. But in our empirical analysis below, we find that most (see Table 1) of the postwar increases in per-capita marriage rates, for women as well as men, were due to sharp increases in the marriage-hazard rates. We establish two key facts: 1) the age-specific marriage-hazard rates of women were 50% higher after the war than before, and 2) it took until the mid-1920s for female hazard rates to return to trend. Thus the post-war response of female marriage hazards is opposite in sign to that predicted by the sex-ratio hypothesis, and this effect persisted over several years.

How are these facts explained in the context of the matching model? More generally, what is the mechanism by which a transient negative shock to marriage incentives results in a marriage-rate boom after the shock passes? Does the analysis reconcile the data with the sex-ratio hypothesis as described above?

It is tempting to view the increase in female marriage-hazard rates as an implication of the post-war abundance of singles caused by war-time disruption of the matching process. But while the abundance of singles can generate higher *per-capita* rates, it does not, under the usual assumptions of constant returns to scale and constant search intensity, explain the marriage-hazards increase. Moreover, violating either of these two assumptions leads to

[1] This decomposition is orthogonal provided that the size of the singles pool has no effect on marriage-hazard rates, which is true under the standard assumption of constant returns to scale in matching.

multiple equilibria, which would be difficult to reconcile with a third important fact about French marriage rates: long-run stability. As can be seen in Figure 1, after each major disturbance, from 1800 to 1950, the per capita marriage rate returns eventually to the long-run average.

We propose an explanation based on war-induced changes over time in the composition of the singles pool. Our idea is that the war-time decline in marriages generated a post-war singles pool with an unusually high abundance of singles with strong tendencies to marry. This composition effect was strong enough to offset the effect on female marriage hazards of the lower post-war male-to-female ratio. Such an effect would arise whenever the war-time marriage-hazard decline is stronger for singles with a higher marriage propensity. Matching frictions and competition across age groups would both serve to propagate the composition effect over time.

We implement this idea in a directed-search matching model where the underlying heterogeneity reflects differences in utility from single life, and among single women, differences in the propensity to have children. To further discipline our approach, we impose that all such differences are age-related. Composition effects can therefore result in higher age-specific female marriage-hazard rates due to shifts in either the male or female composition or both. The focus on fertility is motivated by the war-time birth-rate bust (a 40% decline), and fits well with the standard Becker (1981) view of marriage as an arrangement for the raising of children.

In the context of our model, the war consists of a pair of temporary, unanticipated shocks: higher mortality for younger men, and weaker fertility incentives for married couples. If we assume that the marriage market was in a stationary equilibrium before the war, then the post-war corresponds to the gradual return to this equilibrium. Therefore equilibrium hazard rates evolve both during the war, in response to these shocks, and after the war, due to changes in both the current composition of the singles pool and in expectations about the future composition.

Does this model provide a plausible explanation of the post-war rise in the female marriage hazard? Our general argument suggests that the female marriage hazard could rise or fall, depending on how the type-specific marriage hazards respond during war time. Because the types of singles are not observable from our data sources, we adopt a calibration strategy that proceeds by three steps: 1) calibration of the model so that the stationary equilibrium matches the pre-war marriage and birth statistics, 2) calibration of the wartime shocks so that the calibrated model matches the average war-time observations for the aggregate birth rates and the male mortality rate, 3) computation of the sequence of matching-market equilibria along the transition path back to the steady state. This strategy ensures that the parameters of the model, and hence the elasticities of the hazards with respect to the war time shocks, are pinned down by the peace-time statistics, while the size of the war time shocks can be fixed from observations on aggregate births and mortality rates.

Our strategy does not guarantee that the war-time shocks will generate a war-time marriage-rate decline of the correct magnitude. Although age-specific hazard rates are not available during the war, we are able to infer the average war-time hazard rate from the annual per-capita marriage rate and the population changes in the annual census immediately before and after the war. We find that the response of marriage hazards in the model to the weaker marriage incentives implied by the fertility shock does indeed account for the war-time decline in the marriage-hazard rates for singles aged 20-29, supporting the premise that marriage incentives are driven by anticipated fertility.

Our main result is that the model's transition path replicates to a large extent the post-war patterns of marriage hazards. The model's transition path exhibits: A war-time marriage-rate decline that is roughly equal in size to that observed in the French data, a decline in the male-to-female ratio among singles that matches the empirical value as of 1920, and a large post-war boom in female marriage probabilities; about 63% of the observed peak for women aged 20-39. The model also generates a long transition back to the steady state, accounting for no less than 73% of the half lives of the post-war peaks observed in the data.

Why are female marriage probabilities higher after the war despite the low female-to-male ratio among singles? We were surprised to learn that this had little to do with the composition of female singles. Inspection of the model's output reveals an unusually high post-war abundance, relative to the stock of single women, of men with a high propensity to marry. This male composition effect turns out to be so strong that it outweighs the negative effects of the male mortality shock, via the decline in the male-to-female ratio, on the female marriage hazard.

Why are male marriage probabilities higher after the war? High-propensity men are also unusually abundant, after the war, relative to the stock of single men. This is due in part to the war reducing their marriage rates relative to those of low-propensity men, as well as to the fact that war-time mortality was much higher for younger men, who typically married with lower probabilities. The war-induced composition effect therefore raises men's marriage probabilities, and accounts for 44% and 61% of the peaks of single men in the 20-29 and 30-39 age groups, respectively.

To determine the effect of each shock in isolation, we compute additional transition paths with only one shock each. We find that the male mortality shock alone would have yielded a 23% decline in women's marriage hazard after the war, as standard models predict. This means that the overall peak in female marriage hazards strongly understates the impact of the composition effect. In the absence of the male mortality shock, the composition effect would have generated an 39% post-war increase in female marriage hazards, 33% higher than in the baseline experiment.

What accounts for the unexplained remainder of the post-war marriage peaks? We note that France adopted strongly pro-natalist policies after the war, including strong economic incentives for fertility (see Roberts, 1994). Births per married woman increased sharply relative to trend, a change that persisted well into the 1930s. To what extent could stronger post-war fertility demand explain the residual marriage-hazard peaks? To find out, we add to our war-shock exercise a post-war shock to fertility incentives, calibrated to match the post-war aggregate birth hazard for married couples. The share of the post-war marriage-hazard peaks explained by the model increases significantly; to 61% and 80% for men in the 20-29 and 30-39 age groups, respectively, and to 97% and 88% for women in those age groups. Thus, the model now accounts for essentially all of the anomalous post-war increase in female marriage hazard rates.[2]

In general, our work suggests that the causes of sex-ratio differences should be considered before inferring welfare effects. It is clear from our analysis that the age-specific marriage prospects of single women are much improved post-war, despite the decline in the sex ratio. To the extent that the composition of the singles pool can be assumed to be independent of sex-ratio variations, our results strongly support the sex-ratio hypothesis. However, our results also suggest that this independence condition did not hold with respect to the war in France, and so the sex-ratio effect was swamped by the composition effect.

Relation to existing literature

It is clear from existing research that the post-war marriage cohort was responding to more than just a simple delay in registration of existing matches. Both Henry (1966) and Abramitzky et al. (2011) document that, in post-war marriages, women married men who,

[2]Vincent (1946) ascribes post-war changes in marital fertility as arising in part from the impact of war on marriage rates; our paper provides an alternative perspective: changes in anticipated fertility incentives can explain large fluctuations in marriage rates.

on the basis of observables at least, were less attractive than the husbands acquired by earlier or later marriage cohorts. Henry (1966) found that women in the post-war marriage cohort made up for the short-fall of men by marrying immigrants, widowers, and most of all, younger men.[3] Abramitzky et al. (2011) found that, in areas with higher military mortality rates, women tended to marry lower-status men after the war. Delayed registration of existing matches cannot explain such changes in the composition of marriages. Nor can such substitution, reflecting intenser competition among women for husbands, explain an increase, relative to the pre-war trend, in aggregate female marriage probabilities. Indeed the arguments of Henry (1966) and Abramitzky et al. (2011), being based on the stock of married women years after the war, are silent about the dynamics of the flow of singles into marriage.

A similar argument has been made by Brandt et al. (2009), who study the sorting effect of an exogenous change in cohort size due to the 1958-61 famine in China. They find that members of famine-born cohorts had to marry "non-customary" spouses, due to the relatively small size of the surviving famine-born cohorts. Another cohort-size analysis, Bronson and Mazzocco (2014), is more similar to our work in that they model aggregate marriage rate dynamics; their model links marriage propensities directly to age, which precludes composition effects. Neither paper however examines independent variation in the sex ratio or the role of marriage-market interruptions.

Our model can be seen an extension of Kennes and Knowles (2015) to two-sided heterogeneity, though without the complications of unmarried births and divorce. Our basic assumptions are derived from the canonical Becker (1973) model, but with added features: frictions, fertility, and stochastic aging. As in the empirical marriage-matching literature that derives from Choo and Siow (2006), we assume fully transferable utility within marriage, a defining feature of the Becker model, which is also the basis of the influential "collective-model" approach associated, *inter alia*, with Chiappori et al. (2002). We implement this in a competitive-search framework because this simplifies the computation of transition dynamics and eliminates the multiple-steady-state issue analyzed by Burdett and Coles (1999).

By focusing on the dynamics of the equilibrium model, our paper complements a large literature based on stationary equilibria, such as Choo and Siow (2006), Aiyagari et al. (2000) and Chiappori et al. (2015). In fact, the Choo and Siow (2006) approach can be seen as a one-period version of the matching mechanism in our model, with our search frictions replaced by *iid* taste shocks. The extension of that literature to multi-period matching has,

[3]Henry (1966) also finds that the marriage patterns returned to normal in later female birth cohorts.

however, proven very difficult; Choo (2014) appears to be the first paper to achieve this, but is limited to analysis of the stationary state. Dynamics also play a role in more recent papers, such as Greenwood et al. (2013).

2 Empirical Analysis

2.1 Aggregate Marriage Rates

Consider first the time pattern in newlyweds per capita, as compiled by Mitchell (1998) from French Census and vital statistics data. We take the aggregate marriage rate to be equal to half of this number. Figure 1 reveals a pattern of war-time busts in marriage rates followed by post-war booms; this pattern occurs in varying degrees in each of the wars during this period, including the Napoleonic wars of 1803–1815. These cycles are large in amplitude; while the average marriage rate is remarkably stable at 7.5 per thousand, we see it rise by 70% after the Napoleonic wars and after World War 2, by 25% after the much shorter Franco-Prussian war of 1870, and by 100% after WW1. The troughs that correspond with these wars are of the order of 20% below average for the Napoleonic and Franco-Prussian wars, 30% for World War 2, and more than 66% for WW1.

The effect of war-time shocks on marriage rates is also quite persistent. It takes about 5 years after the Franco-Prussian war of 1870 for marriage rates to return to normal, 10 years after WW1, and 5 years after World War 2. Turning to the birth rates, these remain above trend for 10 years following the Franco-Prussian war and 5-6 years following WW1.[4]

In what follows, we restrict the analysis to France and WW1 for several reasons. First, the timing of war with Germany was unexpected, and the impact on civilian life was severe and relatively uniform over the course of the war. On June 28, 1914 Archduke Franz Ferdinand of Austria was assassinated; as an indirect result of this essentially random shock, Germany began its assault on France little more than a month later. Furthermore, combat in France, including many major battles, continued until the very end of the war.[5] Second, World War 2, the obvious alternative candidate for analyzing the effect of a war, is likely to be a worse

[4]The patterns for World War 2 are much less distinct, perhaps because German occupation resulted in a resumption of peace-time civilian life for most of France.

[5]The last soldier to be killed, an American, was shot through the head at 10:59 a.m. on Nov. 11, 1918, the final minute of the war, in the Argonne area of France. New York Times, December 28, 2014 - "Where the Great War Ended"

alternative. It was better anticipated, and it broke out in an abnormal economic environment caused by the great depression, and an abnormal demographic environment caused by World War 1.[6] Third, the French system of universal conscription dropped all exemptions in 1905, so all men aged 20 and over were subject to military service regardless of marital, parental or economic status. In the U.K. on the other hand, married men were initially exempt from the draft. Fourth, although individual-level records are not available, we have annual population figures for France, by age in years, sex and marital status, starting in 1900; these are not available for the other countries involved in the war. Finally, it is also clear that the pattern of marriage rates for France in WW1 is not atypical; in Figure 2, for example, we see that the WW1 marriage-rate cycle is remarkably similar for Germany and France. During the war the per-capita marriage rate declines to 50% of its pre-war average in Germany, while the post-war rate peaks at 80% above its pre-war average[7].

2.2 Accounting for Marriage Rates

We now turn to an accounting decomposition of the post-war peak in marriage rates. The aggregate marriage rate by age equals, by definition, the product of the hazard rate and the singles ratio:

$$\underbrace{\frac{\text{Flow of marriages}}{\text{Population stock}}}_{\text{marriage rate}} = \underbrace{\frac{\text{Flow of marrriages}}{\text{Singles stock}}}_{\text{hazard rate}} \times \underbrace{\frac{\text{Singles stock}}{\text{Population stock}}}_{\text{singles ratio}}. \tag{1}$$

We will use this identity to compute the share of each of the right-hand variables in changes in the marriage rate on the left-hand side. To the extent that changes in the singles ratio account for the war-time variation in the marriage rates, there is little need for a choice-based model to understand the time pattern; a demographic model with constant matching rates would suffice. However to explain variation in the hazard rates, a model of individual-level decisions to marry would be essential.

To compute marriage hazards we use the Census which lists population by age, sex and marital status as of January 1st each year, starting in 1900, with the exception of the war years 1915-1919. The marital status categories are: never-married, married, widowed and divorced. We combine this data with the war-time aggregate marriage rates compiled by

[6]See Beaudry and Portier (2002) for a discussion of the great depression in France in the 1930s.
[7]Unfortunately, age-specific marriage and birth statistics are not available for Germany or the other principal participants in the war, apart from France.

Mitchell (1998) to identify a time series of marriage hazards by age and sex, from 1906 to 1936; we also estimate a cross-sectional distribution of marriage hazards by age and sex during the pre-war years, which we present in Section 2.5.[8]

Figure 3 shows the time series of marriage hazards by sex for two age groups: 20-29 and 30-39. These age groups account for more than 80% of female newlyweds and 94% of male newlyweds, on average. Three features of these hazard rates that are worth emphasizing are:

1. Marriage hazards decrease during the war.
   It was already clear from Figure 1 that aggregate marriage rates fell to as low as 35% of the peacetime rate. Our time series of hazard rates implies that the average war-time marriage hazard fell more for the 20-29 age group; to 45% and 37% of the pre-war average, for men and women, respectively, while for the 30-39 group the marriage hazards fell only to 86% and 85% of their pre-war averages.

2. Marriage hazard increase above their pre-war averages after the war.
   This is true for each sex and age group. For 20-29 women the increase is 47%; for 20-29 men it is 106%. These figures are 118% and 126% for 30-39 women and men, respectively.

3. Deviations in marriage hazard contribute far more than deviations in singles ratio to the post-war peaks in aggregate marriage rates.
   To see this we compute the decomposition represented by Equation (1) for a typical pre-war year, 1910, and for 1920. Table 1 shows the results. Both the hazard rates and singles ratio are higher in 1920 than in 1910, for each sex and age group. This implies that aggregate marriage rates are higher too. The last line of the table shows, however, that the hazard rates contributed the most (from 63.7 to 92.3%) to the peaks in the aggregate marriage rates.

4. The post-war peaks in marriage hazard persist many years after the war.
   The hazard rate of 20-29 year-old men, for example, in the late 1920s is still 20% above its pre-war average. Along with the (not unrelated) disruption of the sex ratio, this demonstrates that birth cohorts that were too young to be mobilized, including men who were as young as 14 at the outbreak of war, exhibited a significant disruption in marriage patterns long after the war's end.

[8]We provide the details of our procedure in the appendix.

As we noted earlier, the increase in women's hazard rates after the war is intriguing since the male-to-female ratio among singles declined noticeably during the war (see Figure 4).

Could the increase in the female marriage hazard result from a post-war abundance of young widows/divorcees marrying at higher rates? The French data allows us to distinguish unmarried (including widows and divorcees) from never-married individuals. Figure 5 shows that neither the existence nor the magnitude of the post-war peaks in births hazards are significantly affected by this distinction for most age-sex group.[9]

2.3 Birth Hazards

The aggregate birth rate in panel A of Figure 6, as computed from Mitchell (1998), reveals a war-related cycle similar to that of marriage rates. War-time tends to be associated with abnormally low birth rates and the post war with birth-rate booms. During World War 1 the per-capita birth rate fell on average 40% below its trend. We will use this deviation in the per-capita birth rate as a target to calibrate the size of the fertility shock in war-time.

To compute the marital birth hazards in peacetime, we use vital statistics from the French national statistical institute (INSEE), which list the number of live births per woman by age and year, starting in 1901. We combine this with the number of married women from the Census for each age-year group to construct a time series of annual births per married woman for the two age groups. Panel B of Figure 6 shows that marital birth hazards increased substantially in the years immediately following the war: In 1920 they peaked at 66 and 63% above their pre-war trends, by age group. However the birth hazards remained 25% above their trends well into the 1930s. This is a large and persistent shift in marital birth-hazard rates that extends to women too young to be directly affected by the wartime postponement of births.

2.4 Military Mortality

The authoritative source of data on the impact of the war on the French population is Huber (1931), who reports that military losses (killed and missing in action) amounted to 1.4 million men. We infer from Vallin (1973), who shows the number of dead and disappeared by their

[9] For women 30-39 the peak in the hazard rate of unmarried is less pronounced than for never-married. Thus, including unmarried in the analysis does not help in explaining the post-war peaks.

year of incorporation into the army, that older soldiers tended to die at lower rates than young recruits. This pattern implies that the fraction of men of a given cohort who died or disappeared during the war is systematically decreasing with the age this cohort attained in 1914, with the exception of men younger than 20 in 1914.[10] Figure 7 shows this: less than 5% of the men aged 45 in 1914 died or disappeared during the war, 15% of the 35 year old, and 25-30% of the 20-25 year old. This systematic link between age and death rate during the war turns out to be relevant for our argument later in the paper.

2.5 Marriage and Birth Hazards Before the War

We are also able, with the Census data, to compute age profiles for marriage and birth hazards during the pre-war years. In Section 4 we use these profiles to calibrate our model.

Panel A of Figure 8 plots the hazard rates by sex and age, averaged over the pre-war years. There are two important features of these profiles for our analysis. The first is that young men marry at a noticeably lower rate than young women: Men in their early 20s marry with less than a 2% probability in a given year, while that probability is above 6% for women of the same age. The second, and not unrelated feature, is that men tend to marry later than women: The peaks in marriage hazard rates occur around 25 year old for women, and around 30 for men.

Panel B presents the birth hazard for married women, averaged over the pre-war years. This profile is systematically decreasing in age, from about 30% for women in their early 20s, to about 5% for women in their early 40s.

2.6 Summary

In addition to the main facts that we established in this section, namely the central role of marriage hazards in war-related marriage rate variations, and the post-war peaks in female marriage hazards, we also established some secondary facts that will play supporting roles in the theoretical argument to follow. These are the stationarity of per-capita marriage rates over time, the extreme sizes of the war-time marriage and fertility busts, and the disproportionate toll of the war on younger men.

[10] Under the universal conscription law of 1913, military service began at age 20.

3 The Model

The matching process we describe below is derived from the labor-market literature on competitive search with two-sided heterogeneity, such as Shimer (2005). This competitive-search framework is especially suitable for our analysis because it permits us to treat marriage matching as a dynamic process, and hence to compute the transition path for marriage rates. In what follows we describe the model in terms of wage-posting in separate sub-markets, not because we think it is a particularly realistic description of marriage matching, but because it makes the dynamics easier to work with. Because the competitive-search approach implies that the equilibrium assignment equals the unique Pareto-optimal assignment, as demonstrated by Shimer (2005), the algorithm we use to arrive at the equilibrium can be chosen for computational convenience.[11]

3.1 Demography

There is an infinite succession of periods. The population is composed of infinitely-lived men and women, denoted H and F, respectively. Individuals transition through 3 stages of life, denoted by $a \in \{1, 2, 3\}$, as they age; the probability that an individual in stage $a < 3$ transitions into stage $a + 1$ is denoted by $\delta_i(a)$ for $i \in \{H, F\}$. Stage 3 is absorbing. Each period there is an inflow χ_i of new individuals of sex i and stage 1.

Over time individuals in stages $a < 3$ may marry. Marriage is permanent and starts with zero children. Married couples where $a_F < 3$ and $k < K$, where K is the maximum number of children, can choose the probability, π^B, of a birth in the next period. The birth probability generates a disutility $\sigma^F C\left(\pi^B, a_F, k\right)$, where σ^F is a scale parameter. We interpret the disutility $\sigma^F C\left(\pi^B, a_F, k\right)$ as summarizing various tradeoffs associated with attempting to give birth, which we do not model. Thus, the fertility-choice aspect of our model should be seen as a reduced form model that cannot distinguish between preference shocks and cost shocks. In the remainder of the paper we will adopt the terminology "preference for children" when referring to the function C and/or the parameter σ^F.

Single individuals produce utility $y_i^S(s_i)$ each period, where $i \in \{H, F\}$. Marriages produce utility $y^M(k)$ each period, which is perfectly transferable between spouses. Married people like children, that is $y^M(k+1) > y^M(k)$ for $k \in \mathcal{K}$.

[11] In our model the algorithm may affect the distribution of marriage surplus between the spouses, but not the assignment of men to women.

The probability that a single man in stage a_H dies in a given period is $\sigma^D(a_H)$. We assume, for simplicity, that women and married men do not face a mortality risk. We use the probabilities $\sigma^D(a_H)$ and the parameter σ^F to represent the war: Increasing $\sigma^D(a_H)$ will raise male mortality, and increasing σ^F will raise the cost of having children.

3.2 The Value of a Marriage

Let $Y(a_F, k)$ denote the value of a marriage in state (a_F, k). Since $a_F = 3$, is an absorbing state in which women cannot bear children, the value of a marriage in state $(3, k)$ equals the present discounted sum of the utility flow of remaining in the marriage forever: $Y(3, k) = y^M(k) / (1 - \beta)$, where β is the discount factor between periods. Couples with K children are effectively sterile, thus $Y(a_F, K) = y^M(K) / (1 - \beta)$.

For married couples that can have children, the flow utility each period is $y^M(k) - \sigma^F C(\pi^B, a_F, k)$. The value of an (a_F, k)-marriage is the solution of the following fixed point problem:

$$\begin{aligned} Y(a_F, k) &= \max_{\pi^B} \; y^M(k) - \sigma^F C\left(\pi^B, a_F, k\right) \\ &\quad + \beta \delta_F(a_F) E_{\pi^B}\left[Y(a_F + 1, k)\right] \\ &\quad + \beta\left(1 - \delta_F(a_F)\right) E_{\pi^B}\left[Y(a_F, k)\right], \end{aligned}$$

where

$$E_{\pi^B}\left[Y(a_F, k)\right] = \pi^B Y(a_F, k+1) + \left(1 - \pi^B\right) Y(a_F, k).$$

The first part of this objective function is the period flow of utils. The second part is the expected continuation value, conditional on the wife switching to the next stage of her life. The last part is the expected continuation value, conditional on the wife remaining in the current stage of her life.

3.3 Matching and Marriage Hazard Rates

3.3.1 Matching

At the beginning of each period t the population of singles of sex $i \in \{H, F\}$ and state a_i is denoted by $P_{i,t}(a_i)$. Men decide whether to pay a cost ξ to enter the marriage market.[12] We

[12] This stochastic entry cost ensures that some males choose not to participate each period, which considerably simplifies the computation of equilibria.

assume that ξ is an *iid* shock realized at the beginning of each period, before the participation decision is made. The support of ξ is the positive real line, with a cumulative distribution function denoted by Ξ. All single women in states $a_F \in \{1,2\}$ participate in the market.

The marriage market is divided into two sub-markets, one for each state a_F of active single women. Conditional on participating in the marriage market, single men choose which of the two sub-markets they participate in. Matching within a given sub-market occurs through the random assignment of men to women. Women who are assigned at least one man chose a husband from among their suitors. Let the mass of a_H-men who participate in sub-market a_F be denoted $N_t(a_F, a_H)$. Define the length of the queue of a_H-men in this sub-market as

$$\phi_t(a_F, a_H) = \frac{N_t(a_F, a_H)}{P_{F,t}(a_F)}.$$

To ensure the feasibility of the matching assignments, the following resource constraint must be satisfied for each a_H :

$$\sum_{a_F} \phi_t(a_F, a_H) P_{F,t}(a_F) = \mu_t(a_H) P_{H,t}(a_H), \tag{2}$$

where $\mu_t(a_H)$ is the endogenously-determined fraction of stage-a_H men participating in the marriage market. This equation stipulates that all the participating men, on the right-hand side, are assigned either to the stage-1 or stage-2 sub-market. We present the determination of $\mu_t(a_H)$ in Section 3.4.1.

3.3.2 Marriage Hazards Rates of Women

Standard arguments show that the number of men assigned to each woman follows a Poisson process; the probability that there are no suitors in stage a_H for a woman in stage a_F is $e^{-\phi_t(a_F,a_H)}$. Let $\rho_t(a_F, a_H)$ denote the probability that a woman in stage a_F marries a man in stage a_H. Following Shimer (2005), we assume that a woman marries a man in stage 2 whenever at least one suitor is in stage 2; and she marries a man in stage 1 whenever there is at least one suitor in stage 1, and no suitor in stage 2.[13] This implies the following marriage

[13]This rule is optimal when men of stage 2 generate higher surpluses than men of stage 1, as must be the case to rationalize that men of stage 2 marry with higher probabilities.

probabilities:

$$\rho_t(a_F,1) = e^{-\phi_t(a_F,2)}\left(1-e^{-\phi_t(a_F,1)}\right), \quad (3)$$
$$\rho_t(a_{F,}2) = 1-e^{-\phi_t(a_F,2)}. \quad (4)$$

The total number of marriages by a_F-women is $P_t(a_F)\sum_{a_H}\rho_t(a_F,a_H)$. Thus, the marriage hazard rates of these women is

$$\pi_{F,t}(a_F) = \sum_{a_H}\rho_t(a_F,a_H). \quad (5)$$

3.3.3 Marriage Hazards Rates of Men

The number of marriages between men in stage a_H and women in stage a_F equals $\rho_t(a_F,a_H)\,P_{F,t}(a_F)$. This implies that the rate at which men in stage a_H marry in sub-market a_F is

$$\frac{\rho_t(a_F,a_H)\,P_{F,t}(a_F)}{N_t(a_F,a_H)} = \frac{\rho_t(a_F,a_H)}{\phi_t(a_F,a_H)}.$$

The probability that a man in stage a_H marries, conditional on participating, is then

$$\pi_{H,t}(a_H) = \frac{1}{\sum_{a_F}N_t(a_F,a_H)}\sum_{a_F}N_t(a_F,a_H)\,\frac{\rho_t(a_F,a_H)}{\phi_t(a_F,a_H)}. \quad (6)$$

3.4 Value Functions of Single Agents

3.4.1 Men

Let $V_{H,t}(a_H)$ denote the value of a man in stage a_H who has decided to participate in the market, and $R_{H,t}(a_H)$ denote his value of remaining single at the end of the period. Let $v_{H,t}(a_H)$ denote the expected gain from marrying (conditional on participating). We present the determination of $v_{H,t}(a_H)$ in the next section. We have:

$$V_{H,t}(a_H) = R_{H,t}(a_H) + v_{H,t}(a_H). \quad (7)$$

To define the value of remaining single we proceed as follows. First, we note that men in stage a_H participate in the marriage market if and only if $\xi \leq v_{H,t}(a_H)$, that is if and only if their expected gain from marrying conditional on participating is at least as large as the cost of participating. We use the notation $\xi_t^*(a_H)$ to denote the marginal man in stage a_H, that is the man who is exactly indifferent between participation and non-participation:

$$\xi_t^*(a_H) = v_{H,t}(a_H). \tag{8}$$

Thus, the probability that a man in stage a_H participates is $\mu_t(a_H) = \Xi(\xi_t^*(a_H))$. Since ξ is *iid*, $\mu_t(a_H)$ is also the proportion of these men participating in the marriage market at date t. Before the realization of the cost, the ex-ante value of period t for these men is

$$W_{H,t}(a_H) = (1 - \mu_t(a_H)) R_{H,t}(a_H) + \mu_t(a_H) E[V_{H,t}(a_H) - \xi \,|\xi \leq \xi_t^*(a_H)]. \tag{9}$$

The value of remaining single at the end of the period for a man in stage a_H is then

$$R_{H,t} = y_H^S(a_H) + \beta[(1 - \delta_H(a_H)) W_{H,t+1}(a_H) + \delta_H(a_H) W_{H,t+1}(a_H + 1)], \tag{10}$$

where $W_{H,t}(3) = y_H^S(3)/(1-\beta)$. That is, the ex-ante value of being single in stage 3 is the value of remaining a single man forever.

3.4.2 Women

Let $V_{F,t}(a_F)$ denote the value of a woman in stage a_F at the beginning of period t. Let $R_{F,t}(a_F)$ denote the value of remaining single by the end of the period, and $v_{F,t}(a_F)$ denote the expected gain from marrying during the period. We have

$$V_{F,t}(a_F) = R_{F,t}(a_F) + v_{F,t}(a_F). \tag{11}$$

The woman's value of remaining single is defined by

$$R_{F,t}(a_F) = y_F^S(a_F) + \beta[(1 - \delta_F(a_F)) V_{F,t+1}(a_F) + \delta_F(a_F) V_{F,t+1}(a_F + 1)]. \tag{12}$$

This equation states that the value of remaining single by the end of the current period comprises the utility flow from being a single, and a discounted, expected continuation value. The latter depends upon the probability of transitioning into the next stage. For woman in stage 3, the expected gain from marrying is zero since they do not participate in the marriage market by assumption. This implies $R_{F,t}(3) = V_{F,t}(3) = y_F^S(3)/(1-\beta)$.

3.5 Laws of Motion

Given marriage and participation rates, the laws of motion for the population of single men in stage 1 is

$$P_{H,t+1}(1) = (1-\delta_H(1))\left(1-\sigma^D(1)\right)(1-\mu_t(1)\pi_{H,t}(1))P_{H,t}(1) + \chi_H\left(1-\sigma^D(1)\right), \quad (13)$$

that is the number of single men in stage 1 at date $t+1$ is the number of date-t single men in stage 1 who did not transition into stage 2, did not marry, and did not die; plus the flow of new stage 1 single that did not die. For single men in stage 2 the law of motion is

$$P_{H,t+1}(2) = (1-\delta_H(2))\left(1-\sigma^D(2)\right)(1-\pi_{H,t}\mu_t)P_{H,t}(2) + \delta_H(1)\left(1-\sigma^D(1)\right)(1-\mu_t(1)\pi_{H,t}(1))P_{H,t}(1), \quad (14)$$

that is, the number of single men in stage 2 at date $t+1$ is the number of single men in stage 2 who did not transition into stage 3, did marry and did not die; plus single men in stage 1 at date t who transitioned into stage 2 but did not marry and did not die.

For single women in stage 1 these laws can be written as

$$P_{F,t+1}(1) = (1-\delta_F(1))(1-\pi_{F,t}(1))P_{F,t}(1) + \chi_F, \quad (15)$$

and for single women in stage 2

$$P_{F,t+1}(2) = (1-\delta_F(2))(1-\pi_{F,t}(2))P_{F,t}(2) + \delta_F(1)(1-\pi_{F,t}(1))P_{F,t}(1). \quad (16)$$

3.6 Optimality

In each sub-market a_F the women post a utility offer $w_t(a_F, a_H)$ for men in stage a_H. The utility offers are common knowledge and women are able to commit to their offers.

3.6.1 Men

Recall that $\rho_t(a_F, a_H)/\phi_t(a_F, a_H)$ is the probability of marriage in sub-market a_F, for a participating man in stage a_H. Thus, the expected gain from marriage for a man in this sub-market is

$$\tilde{v}_{H,t}\left(a_F,a_H\right)=\frac{\rho_t\left(a_F,a_H\right)}{\phi_t\left(a_F,a_H\right)}w_t\left(a_F,a_H\right).$$

Since participating men can freely choose where to direct their search for a spouse, optimality dictates that in equilibrium they are indifferent between the two sub-markets:

$$v_{H,t}\left(a_H\right)=\tilde{v}_{H,t}\left(1,a_H\right)=\tilde{v}_{H,t}\left(2,a_H\right). \tag{17}$$

3.6.2 Women

The optimal offer $w_t\left(a_F,a_H\right)$ by women in stage a_F maximize the expected gain from marrying, subject to the constraint that participating men must receive their expected values. Thus $v_{F,t}\left(a_F\right)$ is defined by the following optimization problem

$$v_{F,t}\left(a_F\right) = \max_{w_t\left(a_F\right)}\sum_{a_H}\rho_t\left(a_F,a_H\right)\left[x_t\left(a_F,a_H\right)-w_t\left(a_F,a_H\right)\right] \tag{18}$$

$$s.t. \qquad v_{H,t}\left(a_H\right)=\frac{\rho_t\left(a_F,a_H\right)}{\phi_t\left(a_F,a_H\right)}w_t\left(a_F,a_H\right), \tag{19}$$

where the surplus $x_t\left(a_F,a_H\right)$ is defined by the output of a marriage, net of the reservation values of the husband and the wife:

$$x_t\left(a_F,a_H\right)=Y\left(a_F,0\right)-R_{H,t}\left(a_H\right)-R_{F,t}\left(a_F\right). \tag{20}$$

3.7 Equilibrium

A competitive-search equilibrium of our model is a sequence of value functions for men, $\left\{R_{H,t}\left(a_H\right),W_{H,t}\left(a_H\right),V_{H,t}\left(a_H\right),v_{H,t}\left(a_H\right)\right\}$, and women, $\left\{R_{F,t}\left(a_F\right),V_{F,t}\left(a_F\right),v_{F,t}\left(a_F\right)\right\}$, wage offers, $\left\{w_t\left(a_F,a_H\right)\right\}$, participation thresholds, $\left\{\xi_t^*\left(a_H\right)\right\}$, queue lengths, $\left\{\phi_t\left(a_F,a_H\right)\right\}$, and population of single men, $\left\{P_{H,t}\left(a_H\right)\right\}$, and single women, $\left\{P_{F,t}\left(a_F\right)\right\}$, such that at each date t:

1. Men and women are optimizing:
   (a) Women solve problem (18)-(19).
   (b) The marginal man is indifferent between participating or not, Equation (8), and

(c) participating men are indifferent between sub-markets, Equation (17).

2. The assignment of agents to sub-markets is feasible, Equation (2).

3. The population of singles follows the laws of motion in Equations (13)-(16).

4 Calibration and Steady-state analysis

For the quantitative analysis, we begin by first calibrating the model to French pre-war data, and then using the calibrated model to set the size of two war-time shocks, to fertility and male mortality, so that the model matches the aggregate war-time birth and mortality rates. The method we followed for computing the relevant empirical statistics is described in the empirical section above. We then compute the response over time of the model's marriage patterns to these shocks.

Since the match between model and data will not be perfect, we also target the age profile for the sex ratio of singles, as this is a central feature of the theoretical argument. The idea is to avoid deviations in marriage probabilities that are particularly important for matching the sex ratio. To interpret our results in terms of age profiles, we impose that new agents are 18 years old when they first enter the singles pool, and that periods in the model last a year.

4.1 Functional Forms

Our goal in choosing functional forms was to minimize the number of free parameters, subject to allowing a full range of variation in the relevant quantities. For the output of marriage, we chose a form that allowed for diminishing marginal returns to children, with a parameter ω that can be adjusted to vary the intensity of women preferences over children:

$$y^M(k) = \omega \ln(1+k). \tag{21}$$

The birth-probability cost function has two free parameters for each state (a_F, k) in which women can give birth, $\gamma_a(a_F)$ and $\gamma_k(k)$:

$$\sigma_F C(\pi, a_F, k) = \sigma_F \gamma_k(k)\, \pi^{\gamma_a(a_F)}. \tag{22}$$

The distribution of the male participation cost is assumed log-normal with parameters (μ_ξ, σ_ξ), that is $\ln(\xi) \sim N(\mu_\xi, \sigma_\xi)$.

4.2 Optimization over Parameters

A few of the parameters are set to values outside the calibration loop. These are: the annual discount factor that is set to $\beta = 0.96$, a standard value in the macro literature; the maximum number of children per family that is set to $K = 3$; and the inflow rate of men that is normalized to unity, i.e. $\chi_H = 1$.

The remaining parameters are set by a standard non-linear algorithm to minimize the Euclidean distance between the statistical targets and their model counterparts. Let the parameter set be denoted by Θ, that is

$$\Theta = \left(\theta, \gamma_k(k), \gamma_a(a_F), \mu_\xi, \sigma_\xi, \delta_F(a_F), \delta_H(a_H), y_F^S(a_F), y_H^S(a_H)\right).$$

The benchmark parameters are chosen to minimize the distance between model and data:

$$\min_{\Theta} \left\{ \sum_{\tau=20}^{45} \sum_{i=H,F} \left(m_i^\tau(\Theta) - \mathbf{m}_i^\tau\right)^2 + \sum_{\tau=20}^{45} \left(b^\tau(\Theta) - \mathbf{b}^\tau\right)^2 + \sum_{\tau=20}^{45} \left(SR^\tau(\Theta) - \mathbf{SR}^\tau\right)^2 \right\},$$

where $m_i^\tau(\Theta)$ represents the steady-state marriage hazard for people of calendar age τ and sex i. The steady-state birth hazard for women of calendar age τ, is $b^\tau(\Theta)$, and $SR^\tau(\Theta)$ is the steady-state sex ratio for singles of age τ. The empirical target values are denoted by bold face.

We chose these moments because they are informative for the key parameters of the model and, in particular, for those driving the heterogeneity that we postulate across stages. It is important to remember that stages are not observable, but that they are correlated with age via the transition probabilities, $\delta_i(a_i)$. Thus, the age profiles that we target allow us to infer the parameters driving heterogeneity across stages. An exact match to the marriage-hazard targets is unlikely, however. So, we also target the age profile of the sex ratio in order to prioritize matching moments that have a larger influence on the sex ratio.

The mapping from moments to parameters can be understood as follows. The model implies a distribution of stages, at any calendar age τ, for both men and women. These age-specific distributions are dictated by the rate at which men and women transition from one stage to the next. It follows, then, that the height and slope of the age profile for marriage hazards implies values for the marriage hazards of both stages, as well as the change over age of the distribution of each stage. Matching the height of the age profiles of marriage hazard informs the parameters governing the value of single life, $y_i^S(a_i)$. Matching its slope informs, in turn, the transition rates between stages, $\delta_i(a_i)$. Since, in addition, the female stage accounts for all age-related birth-hazard variation, the birth hazard profiles also discipline fertility differences by stage, $\gamma_k(k)$ and $\gamma_a(a_F)$. In this way the heterogeneity in the model is pinned down by the pre-war empirical targets.

4.3 Parameter Values

The resulting benchmark parameter set is shown in Table 2. Men in stage 1 transition with probability 0.12 to stage 2, where they in turn transition with probability 0.05 into stage 3 (inactive men). This means that the average man remains in stage 1 between the ages of 18 and $18 + 1/0.12 = 26$, and in stage 2 between the ages of 27 and $27 + 1/0.05 = 47$. For women, the transition rates of 0.17 and 0.03 imply that the first stage of life lasts, on average, from ages 18 to 24, and the second from ages 25 to 58.

The preference parameters imply that young men get more out of single life than older men do, whereas older women are happier with single life than younger women are. As a result, women will tend to marry earlier than men.

4.4 Match to Statistical Targets

In Figure 9 we compare the age profiles from the model with those from the empirical analysis. Table 3 reports a few model statistics from the figure. While there are localized deviations in the marriage hazards of the young agents, these are relatively small; the age profile for the sex ratio of singles is almost exactly matched. Three important features of the empirical marriage hazard profiles are shared by the model's profiles. First the marriage hazard of men in their early 20s is lower than that of men in their late twenties; Second the marriage hazard of women in their early 20s is higher than that of women in their late 20s; Third the marriage hazard of young women is higher than that of young men, i.e. women tend to marry earlier than men. Allowing for more life stages would permit a better match of the age profile of the marriage hazard. But since the main role of the exercise was to discipline the parameters governing heterogeneity, we leave further refinements to future papers.

4.5 Equilibrium behavior

In the benchmark calibration, the model matches the empirical inverted-u shape patterns of the marriage hazards for men, as seen in the top-left panel of Figure 9. The logic behind this is as follows. First, the model's preference parameters imply, as noted above, that stage-1 men enjoy single life more than stage-2 men do. This is important to generate low marriage hazards at young ages. In fact, the model's implied hazard rate for stage-1 men is zero. All 18-year old men are of stage 1 by assumption, so their marriage hazard is zero. At age 19, a fraction of men, 12 percent to be precise, has transited into stage 2 where they have a positive marriage hazard of 44%. Hence the marriage hazard profile increases between age 18 and 19. At age 20 some men will be in stage 1, some in stage 2, and some will have transited into stage 3. Given the transition probabilities implied by our calibration, $\delta_H(a_H)$, the proportion of stage-2 men at age 20 is large enough to generate an increase in the marriage hazard profile between age 19 and 20. As calendar age increases, however, an increasing proportion of men are in stage 3. This lowers the marriage hazard by age, hence the inverted-u shape.

For women a similar logic applies but the outcome is different since, for women, the high marriage-hazard state comes first. Women marry with a higher probability in stage $a_F = 1$ (18%) than in stage $a_F = 2$ (7%), and then they do not marry at all in stage 3. Thus the age profile for women is declining with age, as seen in the top-right panel of Figure 9. In terms of calendar age, the marriage hazard of women aged 20-29 is 0.12 and of women aged 30-39 is 0.05.

Our calibration also implies that the married birth hazards to stage-2 women are higher than those to stage-1 women; this is required to match the increase of the observed birth hazard with age for young married women, as can be seen in the birth-hazard panel of 9. The birth hazards are declining in the number of children already born. The hazard of first births is 0.291 for married women in stage 1 and 0.367 in stage 2. For mothers of one child, the birth hazards drop to 0.247 and 0.321, respectively. For mothers of two children, the birth hazards drop further, to 0.20 and 0.27, respectively. In terms of calendar age, the birth hazard for women aged 20-29 is 0.25 and of women aged 30-39 is 0.10.

5 The Impact of the War

We now describe the main experiment of this paper. We start from the steady-state equilibrium and assume that two simultaneous, unanticipated shocks of five-year duration hit the economy: a male-specific mortality shock and a fertility shock. We compute the transition path, back to the steady-state, under the assumption that the population responds optimally to the shocks and that agents perfectly anticipate the end of the war, that is when the variables representing the war return to their pre-war values.

We represent the fertility shock via an increase in σ^F(equal to 1 in the steady-state); this is equivalent to a negative shock to preferences for children –see Equation (22). We assume that σ^F increases to the same values for all stages of marriage. The increase is not gradual: σ_F jumps up to its war-time value in period 1 of the transition, remains at this value for 4 more periods, and returns to 1 in period 6, and thus generates a decline in birth hazards throughout the war. We set $\sigma^F = 4.5.$ so that birth rates declines by 50% on average during the war, as reported in the French vital statistics.

We represent male-specific mortality by setting the stage-specific male death rate to $\sigma^D(1) = 0.045$, and $\sigma^D(2) = 0$ for the duration of the war. This value ensures that the model's prediction of the post-war sex-ratio of singles aged 20-39 matches the empirical value as of January 1920, the first-post-war observation in the French Census. Finally, our approach also ensures that the model approximates the age profile of war-time mortality risk, computed from Vallin (1973) which implies that annual military mortality was a steeply declining function of age.

5.1 The War-Time Effects of the Shocks

How do marriage and birth hazards respond to the wartime fertility and mortality shocks? We start with a description of the war-time effect of the shock. As can be seen from Part A of Table 4, war-time marriage hazards by age group are much lower than in the steady-state equilibrium. For men, the average marriage hazards during the war fall to 43% and 47% of the steady-state values. For women aged 20-29, the decline is similar, to 44% of steady-state. Thus, the war-time collapse in marriage hazards predicted by the model matches closely the statistics reported in Section 2 for the most important population group, men and women between 20-29 years old. For older women, the decline is more severe, to 6% of the steady-state hazard, even though the birth-hazard decline is similar for both age groups, about

50%. Since nothing in our procedure forces the model to match the war-time marriage-rate decline, the size of the fertility effect can be seen as support for theBecker (1981) assertion that marriage is mainly motivated by the desire for children, .

The marriage-hazard declines imply larger populations of singles after the war, as indicated in Part B of Table 4. In the model, the population of single women is 32% and 25% higher than steady-state in 1919, for the 20-29 and 30-39 groups, respectively. This compares with 43% and 25% in the data. Thus, the low marriage hazard of the older women does not significantly distort the post-war singles share. For men, the stock of singles exceeds the steady-state by 18% and 28%, for men of ages 20-29 and 30-39, respectively. This compares with 12% and 21% in the data. This increase in the size of the singles pool will of course increase per-capita marriage rates post-war, but has no direct implication for any change in marriage hazards.

The consequence of the war that is critical for our argument is the impact on the composition of the singles pools; for any given age-sex cell, the fraction that are in any given life stage is function of recent marriage hazards, which respond endogenously to the war shocks, and of exogenous transition hazards, which are unaffected by the war.

War-time marriage hazards fall more for women in stage 2 than for those in stage 1 (Table 4). The reason for this is that the calibration implies that for stage-2 women, fertility accounts for a bigger share of the gain from marriage than for stage-1 women; because stage-2 women have a higher autarky value, they marry mainly for the chance to raise children. Given that they also have a much higher probability of transiting to sterility before the war is over, the prospect of waiting five years to have children further reduces their gains from marriage, relative to those of the stage-1 women. This means that the women added to the singles pool by the war will be more likely than women in the steady-state single population to be women with low marriage propensities. Of course stage-1 women will also be among the additional singles, as their marriage hazards fall too, but their share of the post-war singles pool will be further reduced by their high (exogenous) rate of transition into stage-2 women. Therefore, the war changes the composition of the pool of single women in a way that is conducive to *lower* hazard rates than before the war since low-marriage-propensity women become relatively more frequent.

In the case of men, similar effects are at work, but there is one basic asymmetry that is critical for our results; stage-2 men are more likely to marry, while stage-2 women are less likely. This is not by assumption, but rather the result of calibrating the model to match the observation that age profiles for marriage peak later for men than for women, as described above. As a result, the war reduces the marriage hazard of stage-2 men relative to that of stage-1 men, adding stage-2 men to the singles pool. These additional stage-2 men tend to transition over time into low-marriage-rate stage-3 men, but at a fairly low rate. Furthermore, higher war-time mortality reduces the frequency of stage-1 men. In contrast to the female case, the composition of the pool of single men becomes more concentrated in high-propensity men, leading to *higher* hazard rates.[14]

5.2 The Post-War Transition

Our main result so far is that, even though the sex-ratio declined as a result of the war, the number of men with high gains from marriage increased dramatically relative to the number of single women. We first analyze the implications of this for the post-war peaks in marriage and birth hazards and then turn to the persistence of the marriage trends in the years after the war.

In Part 1 of Table 5 we see that immediately after the war, the marriage hazards of men in the 20-29 and 30-39 age groups increase in our model by 47% and 77%, relative to the steady state, respectively. This amounts to 44% and 61% of the empirical differences between the year 1920 and the pre-war averages. For women, the model's transition path generates marriage-hazard increases of 29% and 74%, respectively by age group, amounting in both cases to 63% of the empirical change in 1920 relative to the pre-war averages.

These peaks in marriage hazards result from the changing composition of the pool of singles due to the war. The abnormal abundance of stage-2 men, shown in Panel A of Figure 10, makes it is easier for women to find a match. This raises the marriage hazards of stage-1 and stage-2 women above their steady state values, contributing to the post-war peaks. But, there is also an abnormally low fraction of stage-1 women in the pool of singles just after the war, shown in Panel C of Figure 10. Since these women typically marry at a higher rate, this effect tends to reduce the post-war peaks. Our benchmark calibration implies that the first effect, i.e. the abundance of men with high gains from marriage, is larger than the second effect, and so women's marriage hazards rise sharply when the war ends.[15]

[14]The extent to which the mortality of stage-1 men contributes or detracts from this effect is discussed in the next section.

[15]The low proportion of stage-1 women after the war is due, as discussed earlier, to the decline in the marriage hazard of stage-2 women during the war. We have seen that an implication of this was that the marriage hazard of 30-39 year-old women declined significantly more in the model than in the data. Had the model more closely matched the decline in the marriage hazard of 30-39 year-old women, the composition effect would have been smaller, and this would have been a force toward larger post-war peaks for women.

Men's post-war marriage hazards are also subject to conflicting effects. The abnormally large proportion of stage-2 men per woman shown in Panel A of Figure 10, makes it harder for these men to marry. This reduces both their participation rate and their marriage hazards. But these men also constitute an abnormally large fraction of single men after the war, which tends to increase male marriage hazards (see Panel B of Figure 10). The benchmark calibration implies that this effect dominates, and so men's marriage hazards also rise sharply when the war ends.

Table 5 also shows that birth hazards per couple are a little higher in the immediate post-war period, relative to steady-state; 6% higher for the women aged 20-29 and 13% for those aged 30-39. Again, there are conflicting effects at work here. On the one hand married couples exit the war with fewer kids than they would normally have had, which should increase birth hazards. On the other hand they are more likely to have transited to stage 3, which reduces them. The net overall effect is an increase that is small relative to what we see in the data, an issue we revisit below, in Section 5.4.

Female marriage hazards in the model, as with the empirical hazards, return to the steady-state level around 1924, 6 years after the end of the war. For men, the persistence is even stronger; as late as 1930 marriage hazards for men aged 20-29 remain well above the pre-war rate, as discussed in the empirical section above. Consider the half-life of the marriage-hazard peaks, as reported in Part 2 of Table 5. In the data, it takes 18 months for the marriage hazard of 20-29 year-old men to close half the gap between the post-war peak and the pre-war averages. In the model, the half life is 13 months, about 73% of that in the data. For 30-39 year-old men, the model generates a longer half life than measured in the data (i.e. 114%). For women, the model generates 108% and 82% of the observed half life, respectively.

The reason for this persistence can be seen in Panels A and B of Figure 10. Panel B shows that before 1925, the fraction of men who are in stage 2 remains well above the steady state; male marriage hazards are higher because men in stage 2 marry at higher rates than those in stage 1. This also generates persistence of female marriage hazards. After 1925 however, the proportion of single men who are in stage 2 returns to normal. But in Panel A we see that the ratio of these men to single females starts falling in 1922, declining eventually to 80% of the steady-state level. This raises the marriage hazard of men in stage 2; this explains the persistence of male marriage hazards but has a negative impact of female marriage hazards.

Thus, we view the mismatch between the model-generated marriage drop for 30-39 year-old women and the data as a "conservative" error acting against the formation of post-war peaks.

Our model is consistent therefore with two persistence facts: marriage hazards remain higher for both sexes years after the war, and male hazards persist much longer than female. The first is explained by the abnormal composition of the pool of singles, the second by the more traditional marriage-squeeze mechanism. We conclude that the transition path of our model succeeds in generating both significant post-war marriage-hazard peaks and in reproducing their persistence.

5.3 Analysis

The cause of the war-time marriage bust is not the subject of our paper. Yet, it is interesting to see that although the context of mobilization and combat suggests an extreme disruption of civilian life, the two observable variables, birth rates and male-mortality, turn out to be sufficient to explain all of the war-time marriage bust, without recourse to an increase in unobservable matching frictions. But what are the contributions of these two shocks, taken one at a time? To answer this question we present now the results of two computational experiments. Each consists of recomputing the transition path with one shock only.

We proceed by revisiting the impact of the war on the marriage hazards. In Figure 11 the marriage hazard patterns are broken out for each of the three experiments: baseline, war-time mortality shock only, and fertility shock only. It is very clear from the figure that the fertility-shock experiment results track the marriage hazards from the baseline experiment much more closely than the war-time mortality shock results do. We infer from this that the fertility shock is much more important for understanding the impact of the war on marriage patterns, both during and after the war.

In percentage terms, the impact of each shock on the size of the post-war marriage-hazard peaks is shown in Part 1 of Table 6. The fertility-shock experiment generates 77% of the baseline post-war peaks for the younger age group of men, and 80% for the older group. For women, the corresponding impacts are 134% and 128%.[16] In the mortality-shock experiment, on the other hand, women's marriage hazards fall by 23% and 18%, respectively by age group. This confirms that the mortality shock in our model is very strong; our assumptions have not generated post-war peaks in female marriage hazards by assuming away the impact of this shock. Of course this shock drives the marriage hazards of women in the wrong direction, but that makes it all the more remarkable that the baseline model can still generate such large post-war peaks in the female marriage hazard.

[16] The effect of the fertility shock on female post-war marriage hazards is larger than in the baseline, because these are reduced by the mortality shock.

To understand why the fertility-shock effect is so strong, we now revisit the impact of the war on the composition of the singles pool. In Panel A of Figure 11 we show for each experiment the time path for the ratio of stage-2 men per woman. This is the composition variable explains the post-war peaks in female marriage hazards. It turns out that the war-time mortality shock causes a 20% decline in the ratio of stage-2 men per woman, by removing stage-1 men who would have transited during the course of the war into stage 2 men. The fertility shock on the other hand causes the ratio to increase by 55%, so that in the baseline experiment this ratio is 40% higher at war's end than in the steady state, generating the rise in the female marriage hazard.

While Panel A clearly accounts for the female-marriage hazard peak, the higher ratio of stage-2 men per woman also implies a decline in the marriage hazard for stage-2 men. The post-war peak in male marriage hazards is explained in Panel B of the same figure, which shows the time path for the fraction of single men who are in stage 2. It is apparent that the fertility shock alone generates an increase of nearly 80% in this fraction, due to the drop in marriage rates associated with the fertility shock during the war, while the mortality shock causes only a 20% increase. Thus, while the mortality shock also contributes to the male post-war marriage peak by increasing the concentration of stage-2 men in the male singles pool, it is far from being the main cause.

This last result is all the more surprising when we consider Panel E of Figure 11, which shows that the sex ratio of singles declines sharply over the course of the war, from 1.37 to 1.1, and that this is largely due to the mortality shock. It would be tempting to conclude that the post-war male marriage-hazard peaks are explained by the fall in the sex ratio and natural therefore to attribute this to male wartime mortality. But we know from Table 6 that this would be highly misleading: only 13% of the baseline marriage-hazard peak is accounted for by the mortality experiment, compared to 77% for the fertility shock.[17] The mortality effect is weak because mortality risk during the war was a steeply declining function of age, and younger men are far less likely to marry. The changing composition of the singles pool is the key to understanding the post-war peaks in the marriage hazards, and this composition responds to the collapse of marriage during the war much more than to male mortality.[18]

[17]We infer from this that interaction among the two effects accounts for the remaining 10%.

[18]The mortality effect is much more important for persistence; Panel A of Figure 11 shows that the eventual decline of the ratio of stage-2 men to single women, and hence the higher marriage rates of the younger men as late as 1930, is entirely driven by the war-time mortality shock.

5.4 Marriage and Pro-Natalism

Can other shocks help our model account for the unexplained portion of the post-war increase in the female marriage hazards? Consider the evidence for a post-war shock to fertility preferences. Figure 6 shows a post-war shift to a significantly higher time path for the rate of post-war births per married woman. This rate remained 30% above the pre-war trend well into the 1930s. Our baseline results instead show only a much smaller birth-rate shift after the war, amounting to about 8% of the empirical increase for women aged 20-29.

Since the war of 1870, a strong pro-natalist movement in French politics had rallied around the fear that France's low population growth, relative to Germany's, was undermining the ability of France to defend itself militarily in the long run. This movement thrived after the German invasion in World War 1, to the point that by 1920, the pro-natalist movement was sufficiently powerful that the Conservative government saw fit to impose significant financial incentives to increase birth rates, as well as legislation against abortion and contraception. At the same time, employers were pressured to offer family allowances to workers with more than two children.[19] Although we can't distinguish the effects of this campaign from the potential impact of other forces outside our model, it's plausible that post-war natalism explains the elevated rate of post-war births per married woman.

In this section, we take as given the higher birth propensities of the post-war period and ascribe them to a permanent 'pro-natalist' shock, realized at the start of the war. We ask whether this shock can account for the portion of the post-war marriage hazard peaks that remain unexplained in our model. The idea is that higher post-war fertility should increase the gains from marriage and therefore increase marriage hazards. The war-time and post-war values of the fertility preference parameter are now set so as to (1) reproduce the 50% decline in the per-capita birth rate during the war, as was done in the baseline experiment; and (2) reproduce a 30% increase of fertility above steady-state after the war.

The results, shown in Table 7, confirm the importance of the fertility shock. The share of the post-war marriage-hazard peaks accounted for by the model increase to 97% of the empirical increase for women aged 20-29, and 88% for women aged 30-39, relative to 63% for both groups in the baseline experiment. For men, this version of the model accounts for 61% and 80% of the post-war peaks, respectively, compared to 44% and 61% in the baseline.

[19] *La loi d'encouragement aux familles nombreuses* was passed in 1923; incentives included maintenance payments to large families, baby bonuses, reductions in the price of bread and train fares for families of three or more children, and social assistance to defray the cost of child care. In the 1920s, the *Medaille de la famille Francaise* was awarded to mothers of large families. As described in Roberts (1994), several of these measures pre-date the passage of the bill.

How does this work? Because single agents anticipate the higher value of a post-war marriage due to the post-war fertility hike, the reservation value of singles in war time remains higher than in the baseline experiment. The resulting war-time decline in marriage hazards is even more pronounced in this experiment than in the baseline. The marriage hazards in the 20-29 age group fall to 38% for men (compared to 43% in the baseline), and for women to 38% (compared to 44%). In addition to the direct effect of higher fertility on post-war marriage, this bigger war-time disturbance exacerbates the composition effects that we described in Section 5.2, hence the larger response of post-war marriages to the war.

We interpret our results as supporting the natalist fear that the fertility of French couples was not going to spontaneously rise to replace the war-time shortfall in births, much less the additional shortfall in births occasioned by military mortality. The pro-natalist program, to the extent that it succeeded in raising the birth rate of married couples, also strengthened incentives to marry, resulting in a further increase in post-war births. In the context of this higher birth rate, driven by stronger incentives, whether pecuniary or hedonic, to have children, our model now explains virtually all of the post-war increase in female marriage hazards in the 20-29 age group.

6 Conclusion

The relationship between the male-to-female ratio and the marriage prospects of singles is central to theoretical and empirical studies of marriages. A low male-to-female ratio is expected to reduce women's marriage probabilities and increase that of men. Using a historic episode of a large change in the male-to-female ratio that was both exogenous and unanticipated, we document an anomalous pattern: a 33% decline in the male-to-female ratio (in the age 20-29 group) appears to result in 50% increase in female marriage probabilities. The effect is remarkably persistent over time and is associated with abnormally high marital birth and male marriage hazard rates that persist into the 1930s.

Our hypothesis attributes this episode to the impact of the war-time marriage decline on the post-war composition of the singles pool. Our quantitative results suggest that roughly 2/3 of the post-war peak in female marriage hazards in the 20-29 age group was due to an unusual abundance of highly-marriageable men, induced by the war-time marriage bust.[20]

[20] Our hypothesis accounts also for the persistence of higher male marriage hazards, even among men who were too young to have been directly affected by the war, as competition among the birth cohorts that were mobilized spills over into the younger singles pool.

Post-war pro-natalism appears to accounts for the remainder. While previous research has focused on war-induced mortality and its implications via the sex ratio, we find that the war-time disruption of the marriage market was far more consequential for aggregate post-war marriage patterns.

Equilibrium analysis was crucial for these conclusions. We developed a dynamic model of two-sided matching with stochastic aging. This permitted two important features of our analysis: we calibrated our model to marriage and births data at annual age frequencies, and we computed transition paths by year. Thus the equilibrium aspect not only disciplines, via the calibration procedure, the steady-state composition of the singles pool, it also endogenously generates its time path after the war. The post-war composition was generated from the optimal response of marriage hazards to the endogenous time path of the singles pool, something that would have been impossible to deal with in a reduced-form setting, such as a Markovian demographic model or a regression-based approach where marriage hazards were exogenous.

Our results should not be taken to imply that the consequences of war-time male mortality for marriage-rate trajectories was trivial; indeed, as we pointed out earlier, the wartime bust in fertility may have been *caused* by anticipations of male mortality. We show that the effect of war-related male mortality, *in isolation*, would have reduced female marriage hazards by 23% post-war. Surprisingly, we also found that this would explained very little of the post-war increase in male marriage rates. Our main point is rather that the decline in the sex ratio of singles should be interpreted with caution; to the extent that variations in the *size* of the singles pool are likely to be accompanied by variations in the *composition* of the singles pool, inferences from the sex ratio to marriage-market conditions will in general be unreliable.

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Table 1: Pre- and Post-War Statistics

	Men		Women	
	20-29	30-39	20-29	30-39
Year 1910				
Hazard	0.100	0.096	0.152	0.058
Single frac.	0.682	0.203	0.426	0.161
Agg. mar. rate	0.068	0.020	0.065	0.009
Year 1920				
Hazard	0.205	0.252	0.236	0.117
Single frac.	0.724	0.232	0.548	0.175
Agg. mar. rate	0.149	0.058	0.129	0.021
Fraction of change due to hazards	0.923	0.877	0.637	0.893

Table 2: Calibrated Parameters

1	Demography	
	Discount factor	$\beta = 0.96$
	Men transition	$\delta_H(a_H) = (0.12, 0.05)$
	Flow of stage-1 men	$\chi_H = 1.00$
	Women transition	$\delta_F(a_F) = (0.17, 0.03)$
	Flow of stage-1 women	$\chi_F = 1.06$
2	Single	
	Men output	$y_H^S(a_H) = (19.02, 1.62, 0.00)$
	Women output	$y_F^S(a_F) = (-0.03, 0.44, 2.55)$
3	Participation cost	
	Mean	$\mu_\xi = 3.82$
	Std. dev.	$\sigma_\xi = 0.57$
4	Marriage	
	Output param.	$\omega = 7.76$
	Cost elast.	$\gamma_A(a_F) = (2.43, 2.83)$
	Cost const.	$\gamma_K(k) = (109.47, 81.92, 58.73)$

Table 3: Steady State Statistics

	Men		Women	
	20-29	30-39	20-29	30-39
Marriage rate	0.09	0.09	0.12	0.05
Birth rate	-	-	0.25	0.10
Fraction of single	0.69	0.25	0.47	0.20
Sex ratio of single	1.40	1.16	1.0	1.0

Table 4: Wartime Changes

		Men		Women	
		20-29	30-39	20-29	30-39
1	Wartime shifters				
	Marriage hazard	0.43	0.47	0.44	0.06
	Birth hazard	-	-	0.49	0.49
2	Post-war (1919) singles fraction				
	Data / trend (%)	112	121	143	125
	Model / steady state (%)	118	128	132	125

Note: Part 1 of the table reports the mean value of a variable during the war to its steady-state value. The figure 0.43 for the marriage rate of 20-29 men, for instance, means that the marriage hazard of men during the war was, on average, 43% of its pre-war average.

Table 5: Post-War Changes in the Baseline Experiment

		Mar. Haz. Men		Mar. Haz. Wom.		Birth Haz.	
		20-29	30-39	20-29	30-39	20-29	30-39
1	Post-war deviation from trend						
	Data (%)	106	126	47	118	66	63
	Baseline (%)	47	77	29	74	6	13
	Baseline / Data (%)	44	61	63	63	8	21
2	Months to 50% of peak						
	Data	18	18	9	15		
	Baseline	13	21	10	12		
	Baseline / Data (%)	73	114	108	82		

Table 6: Experiments

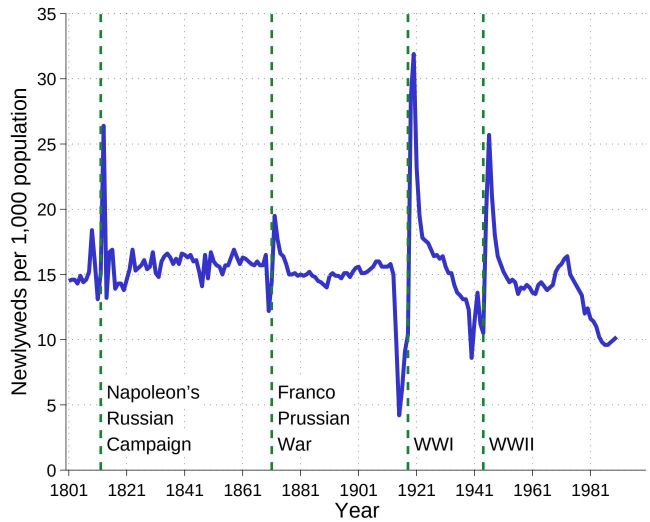
	Mar. Haz. Men		Mar. Haz. Wom.		Birth Haz.	
	20-29	30-39	20-29	30-39	20-29	30-39
1 Post-war deviation from trend						
a. Fert. shock (%)	36	62	39	95	6	19
Exp. / Baseline (%)	77	80	134	128	100	146
b. Death shock (%)	6	6	-23	-18	0	3
Exp. / Baseline (%)	13	8	-79	-24	0	23
2 Months to 50% of peak						
a. Fert. shock (%)	14	21	18	20		
Exp. / Baseline (%)	107	100	180	167		
b. Death shock (%)	10	14	-	-		
Exp. / Baseline (%)	77	67	-	-		

Note: Part 1 of the table reports the post-war deviations from steady state for the marriage and birth hazards in two counterfactual experiments. In the "Fert. shock" experiment the war is represented by the negative shock to fertility incentives, without increases in mortality; in the "Death shock" experiment the war is represented by the increase in mortality alone, without the negative shock to fertility incentives. The row "Exp. / Baseline" reports the ratio between the post-war deviation in a given experiment and the corresponding deviation implied by the baseline model (see Table 5). Part 2 of the table reports the results of these experiments for the persistence of the post-war deviations.

Table 7: Post-War Deviations from Trend in Model with Post-War Pro-Natalist Policy

	Mar. Haz. Men		Mar. Haz. Wom.		Birth Haz.	
	20-29	30-39	20-29	30-39	20-29	30-39
Data (%)	106	126	47	118	66	63
Baseline (%)	65	101	46	104	50	0.2
Baseline / Data (%)	61	80	97	88	76	0.2

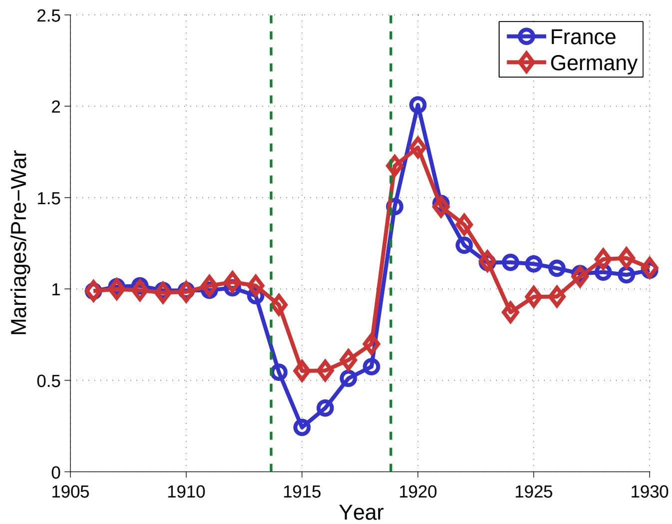
Figure 1: Aggregate Marriage Rate



Source: Mitchell (1998)

Notes: The vertical lines mark the last year of a conflict.

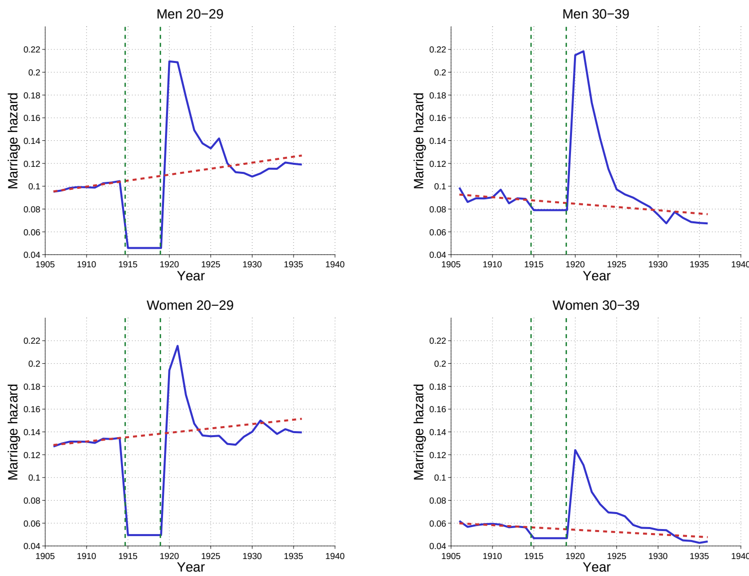
Figure 2: Marriage Rates in France and Germany



Source: Mitchell (1998)

Note: Marriage rates are normalized by their pre-war (1906-1914) averages.

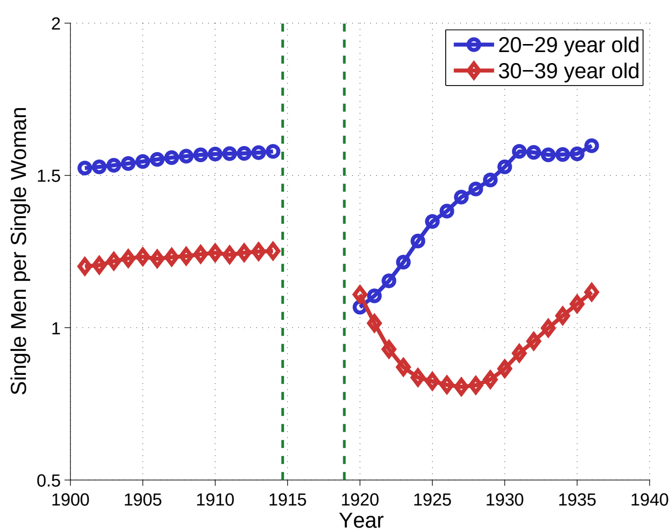
Figure 3: Marriage Hazards by Age and Sex, France



Source: INSEE and authors calculations.

Notes: The vertical lines represent the beginning and end of World War 1. The solid lines represent the hazard rate for a given year, that is the ratio of the flow of marriages in a given year to the beginning-of-year number of single in each age-sex population group. The dotted lines represent pre-war time trends.

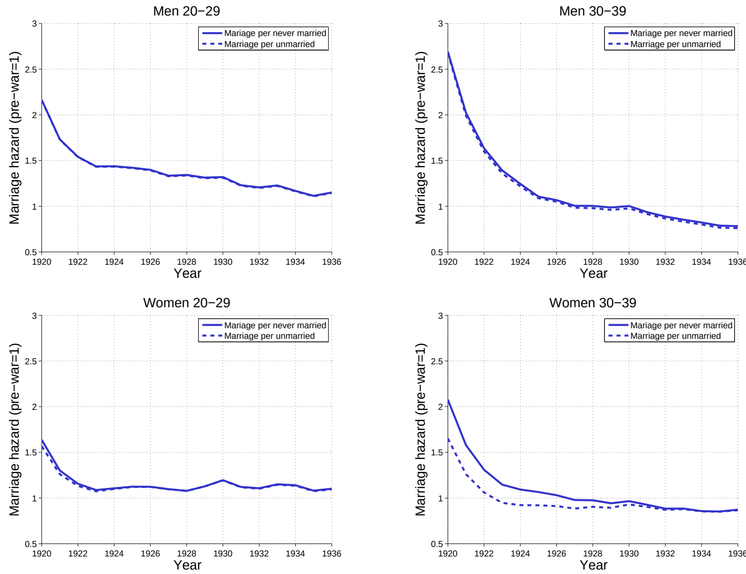
Figure 4: Single Men per Single Women



Source: INSEE

Notes: The vertical lines represent the beginning and end of World War 1.

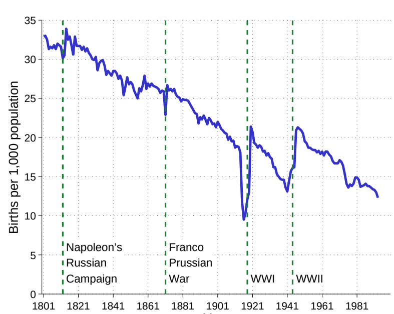
Figure 5: Postwar Hazards by Age and Sex for Never-married and Unmarried, France



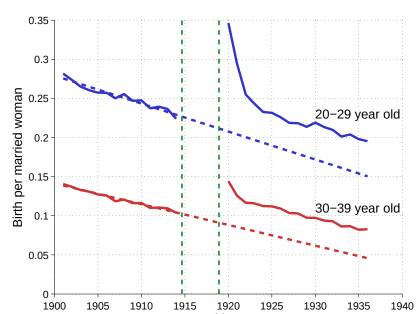
Source: Bunle (1954), INSEE and authors calculations.

Notes: The hazards are computed by dividing the flow of marriages for an age-sex group in a given period, as reported by Bunle (1954), by the stock of never-married, or unmarried at the begining of the period, as reported in the Census. The figure are normalized by their pre-war averages.

Figure 6: Birth Hazards of Married Women



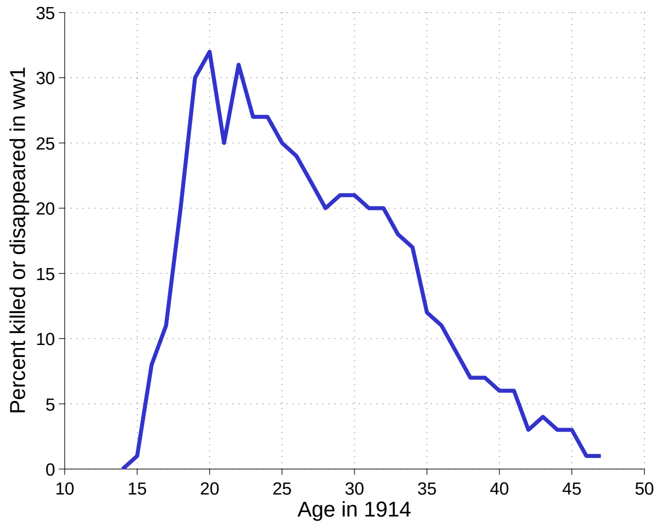
A - Aggregate Birth Rate



B - Birth Hazard of Married Women

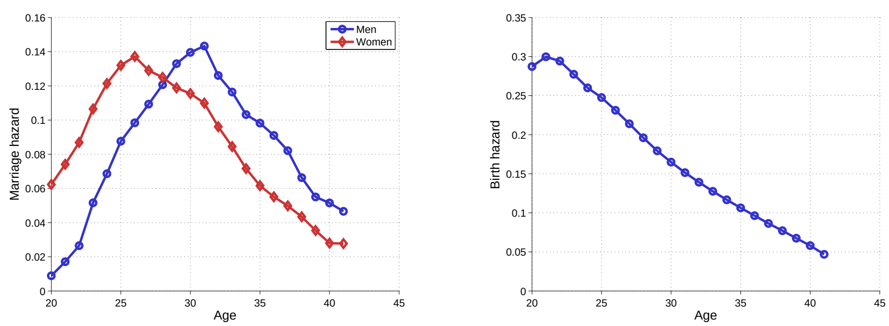
Source: Mitchell (1998), INSEE and authors calculations.

Figure 7: Dead and Disappeared by Birth Cohorts



Source: Vallin (1973) and authors calculations.

Figure 8: Pre-War Marriage and Birth Hazards

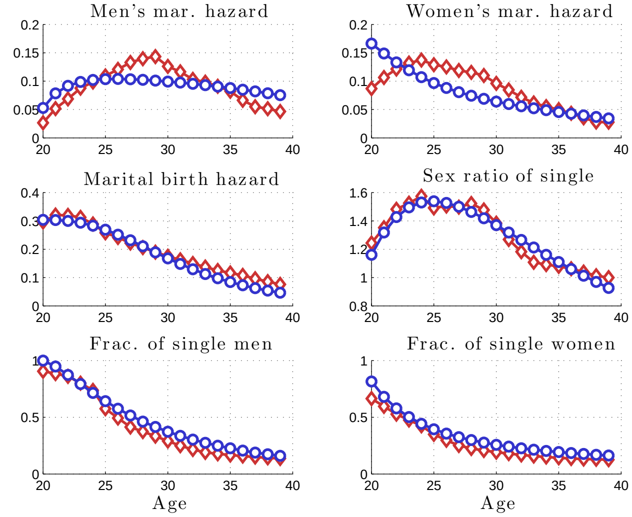


A - Marriage Hazards

B - Birth Hazards of Married Women

Source: INSEE and authors calculations.

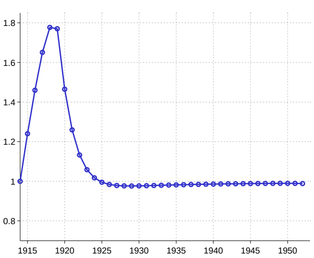
Figure 9: The Model's Fit



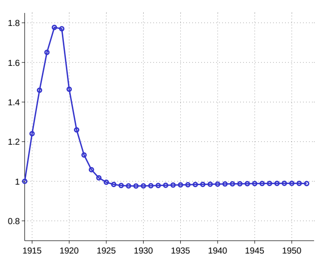
Source: INSEE and authors calculations.

Note: The red diamond are data, the blue circles are the model results.

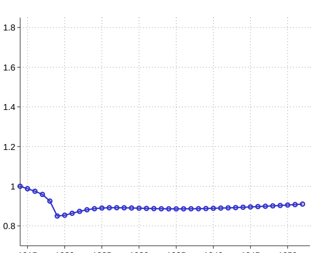
Figure 10: Simulated Single's Population



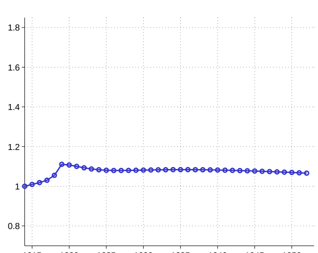
A - Stage-2 Men per Woman
Relative to Steady-State



B - Proportion of Stage-2 Men per Single Man
Relative to Steady-State

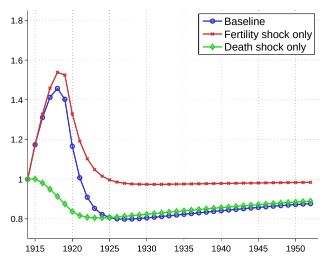


C - Proportion of Stage-1 Women per Single Woman
Relative to Steady-State

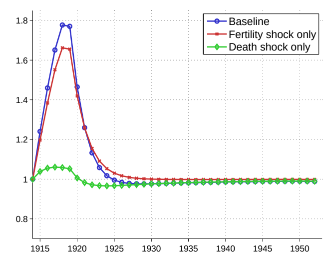


D - Proportion of Stage-2 Women per Single Woman
Relative to Steady-State

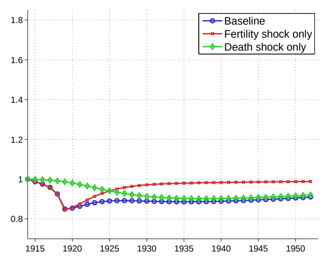
Figure 11: Experiments



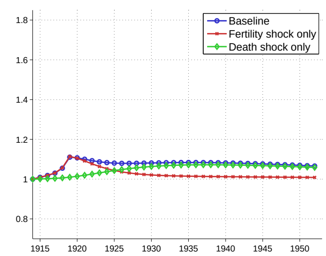
A - Stage-2 Men per Woman
Relative to Steady-State



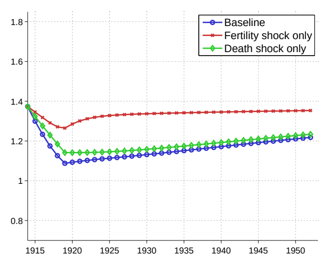
B - Proportion of Stage-2 Men per Single Man
Relative to Steady-State



C - Proportion of Stage-1 Women per Single Woman
Relative to Steady-State



D - Proportion of Stage-2 Women per Single Woman
Relative to Steady-State



E - Sex Ratio of Single

A Computation of Marriage Hazards

To compute age-specific hazard rates, we fit a vector of demographic processes to each year of population data from the French national statistical institut (INSEE). These processes are: marriage, divorce, death, and immigration (net inward). We jointly estimate these for each sex from the annual change in the number of total men and women of each annual birth cohort. For each year from 1905 to 1935, excluding 1914-1919, we apply the method described below, essentially accounting for the exact change in population under a set of simple assumptions. For the war years, 1914-1919, we lack population data, so we apply a modified procedure that exploits information about aggregate marriage rates, military deaths and the population of Alsace-Lorraine, which was annexed to France in December 1918.

A.1 The Peace-Time Computations

Let the total population at time t of people of sex $i \in \{H, F\}$ born in year b be denoted $N_{it}^{b}(T)$, where T stands for "Total." Movement over time in the size of this birth cohort is assumed to be due to either death or net immigration of foreigners to France. At any time $t > b$ we can write

$$N_{i,t+1}^{b}(T) = \left(1 - \pi_{it}^{D}(a) + \pi_{it}^{I}(a)\right) N_{it}^{b}(T),$$

where $a = t - b - 1$ is the age of the birth cohort in years at the start of year t, and $\pi_{it}^{D}(a)$, $\pi_{it}^{I}(a)$ represent annual average death and immigration rates, respectively.

We assume that these two rates are independent of marital status, and that immigration of a married person entails the simultaneous immigration of her spouse. Let $N_{it}^{b}(m)$ represent the population at time t of people of sex i born in year b and currently in marital status $m \in \{M, D, S, W\}$, where M represents marriage, D divorced, S never married and W widowed.

The population of female widows evolves according to:

$$N_{F,t+1}^{b}(W) = \left(1 - \pi_{Ft}^{D}(a) + \pi_{Ft}^{I}(a)\right)\left[N_{Ft}^{b}(W) + N_{Ft}^{b}(M)\,\pi_{Ht}^{D}(a)\right].$$

Similarly for male widows we have :

$$N_{H,t+1}^{b}(W) = \left(1 - \pi_{Ht}^{D}(a) + \pi_{Ht}^{I}(a)\right)\left[N_{Ht}^{b}(W) + N_{Ht}^{b}(M)\,\pi_{Ft}^{D}(a)\right]$$

Note that we have ruled out remarriage within the same calendar year of the spouse death, as well as death of both spouses within the year, and the possibility that widowhood occurs in the same year as the marriage. These events undoubtedly occur but are of low frequency, under the assumption of independence, and even lower if the first event reduces the probability of the second. Together, the above equations form a system that can be used to identify all four rates from the changes in these four population categories.

We now apply the same logic to marriage and divorce rates, taking as given the immigration and death rates. The population of married people of sex i evolves according to:

$$N_{i,t+1}^{b}(M) = \left(1 - \pi_{it}^{X}(a) + \pi_{it}^{I}(a)\right)\left[\pi_{it}^{M}(a)\,N_{it}^{b}(U) + N_{it}^{b}(M)\left(1 - \pi_{it}^{D}(a)\right)\right],$$

where $N_{it}^{b}(U) \equiv N_{it}^{b}(D) + N_{it}^{b}(S) + N_{it}^{b}(W)$ represents the population of unmarried people of sex i in the birth cohort at time t, $\pi_{it}^{M}(a)$ represents the marriage hazard rate of singles, and $\pi_{it}^{X}(a)$ their divorce hazard rate.

Note that we have ruled out remarriage within the calendar year as a divorce, as well as marriage followed by divorce within the calendar year. To the extent that each of these events has a low probability, the chances of both occurring together in the same calendar year are quite small. If, in addition, one of these events temporarily reduces the probability that the second will follow, then the omission is even smaller.

The population of never-married people of sex i evolves according to:

$$N_{i,t+1}^{b}(S) = \left(1 - \pi_{it}^{D}(a) + \pi_{it}^{I}(a)\right)\left(1 - \pi_{it}^{M}(a)\right) N_{it}^{b}(S).$$

Since the divorce rate does not enter this last equation and the death immigration rates are known, then this equation identifies the marriage hazard rate, and the previous equation the divorce hazard rate.

Despite the simplifying assumptions we made about the life processes, our method generates a perfect match to the peacetime data for the following year. This because we are using the next-year data to compute the processes. This ensures that the marriage and divorce rate age profiles, which we will use to create targets in our calibration, are consistent with the population data. The targets themselves are averages over these rates for the 1900-1913 period.

A.2 The War-Time Computations

Because war-time population data is not available, we assume that marriage rates during the war are shifted by a set of shocks $\left\{\omega_{iW}^{M}\right\}$ that remain the same during the war. We compute the war-time mortality rates, as described in the Empirical section above. All other processes are assumed to remain the same as the average for 1910-1913, as derived from the peace-time procedure described above.

The period where the population is not observed extends from January 1915 to January of 1920. This means that we are also missing one-post-war year. So for that year, we assume another set of values $\left\{\omega_{iP}^{M}\right\}$ for the marriage shocks.

The index $i \in \{1, 2, 3, 4\}$ refers to the following demographic groups: men under age 30 ($i = 1$), men of ages 30-39 ($i = 2$), women under age 30 ($i = 3$), and women of ages 30-39 ($i = 4$). Given the shifters then, we can compute the population in 1920 by starting from the population at the start of 1914, and applying the age-specific population processes described above consecutively until we arrive at the population for 1920.

The predicted population under this method requires two modifications. First the population of France in the post war was increased 4% by the addition of Alsace-Lorraine. We increase the 1919 population by 4% in our calculation (implicitly assuming that the composition of the Alsace-Lorraine population was the same as that of the rest of France). Second we know the military deaths by birth cohort (Huber (1931)), so we subtract these from the male population by computing a cohort-specific death hazard rate for each year the cohort was aged 20 or more years.

We choose these 8 shifters simultaneously so as to minimize a score function that consists of the weighted sum of squared deviations from the age distributions of never-married and currently married men and women, plus the difference between the observed per-capita marriage rate (Mitchell (1998)) and that predicted by the shifters.

The resulting shifters are shown in Table 8. Our results imply that the war affected younger people's marriage rates more than those of the older group and women's more than men's. We use the war-time shifters as targets for the calibration of the war shocks in our transition path and assess the ability of the model to match the post-war shifts shown in the table. In this way we are assured that our marriage hazards are roughly consistent with both the aggregate marriage rate and the population changes observed over the course of the war.

Table 8: War-time and post-war shifters for marriage hazards

	Men		Women	
	20-29	30-39	20-29	30-39
Wartime	0.45	0.86	0.37	0.80
Post-war	2.06	2.26	1.47	2.18

B The Value of Marriage and Fertility

In this section we show how we solve for fertility in the steady state and over the transition path.

Guess a the value of a marriage of type (a_F, k): $Y(a_F, k)$. Call this guess λ. The first-order condition for the birth probability, π, is

$$\begin{aligned} C_3(a_F, k, \pi) &= \beta\delta_F(a_F)\left[Y(a_F+1, k+1) - Y(a_F+1, k)\right] \\ &\quad +\beta(1-\delta_F(a_F))\left[Y(a_F, k+1) - \lambda\right] \\ &\equiv mb(\lambda), \end{aligned}$$

where $mb(\lambda)$ stands for the marginal benefit of the birth probability. Assume, as in our calibration, that C is of the form $C(a_F, k, \pi) = \gamma_k(k)\pi^{\gamma_a(a_F)}$, then

$$\pi(\lambda) = \left(\frac{mb(\lambda)}{\gamma_k(k)\gamma_a(a_F)}\right)^{1/(\gamma_a(a_F)-1)}.$$

Now, solve the non-linear equation

$$\begin{aligned} \lambda &= y^M(a_F, k) - C(a_F, k, \pi(\lambda)) \\ &\quad +\beta\delta_F(a_F)E_\pi\left[\pi(\lambda)Y(a_F+1, k+1) + (1-\pi(\lambda))Y(a_F+1, k)\right] \\ &\quad +\beta(1-\delta_F(a_F))E_\pi\left[\pi(\lambda)Y(a_F, k+1) + (1-\pi(\lambda))\lambda\right]. \end{aligned}$$

for λ.

B.1 The War

We assume that the war breaks out unexpectedly in period 1, and lasts for N periods. The end of the war is perfectly anticipated. Let $Y_t^{WAR}(a_F, k)$ denote the value of a marriage during the war, at date t. The value of a sterile marriage with k children during the war is $Y_t^{WAR}(3, k) = y^M(k)/(1-\beta)$ for all war years. Similarly, married couples with K children are sterile, thus $Y_t^{WAR}(3, K) = y^M(K)/(1-\beta)$ for all war years.

B.1.1 Last Year of War

During the last period of the war, the value of a marriage is

$$\begin{aligned} Y_N^{WAR}(a_F, k) &= \max_{\pi^B} \; y^M(k) - \sigma^{WAR} C\left(\pi^B, a_F, k\right) \\ &\quad + \beta \delta_F(a_F) E_{\pi^B}\left[Y(a_F + 1, k)\right] \\ &\quad + \beta(1 - \delta_F(a_F)) E_{\pi^B}\left[Y(a_F, k)\right] \end{aligned}$$

where σ^{WAR} is the wartime value of σ^F, and where

$$E_{\pi^B}\left[Y(a_F, k)\right] = \pi^B Y(a_F, k+1) + \left(1 - \pi^B\right) Y(a_F, k).$$

B.1.2 Previous Years of War

During year $n < N$ of the war we have

$$\begin{aligned} Y_n^{WAR}(a_F, k) &= \max_{\pi^B} \; y^M(k) - \sigma^{WAR} C\left(\pi^B, a_F, k\right) \\ &\quad + \beta \delta_F(a_F) E_{\pi^B}\left[Y_{n+1}^{WAR}(a_F + 1, k)\right] \\ &\quad + \beta(1 - \delta_F(a_F)) E_{\pi^B}\left[Y_{n+1}^{WAR}(a_F, k)\right] \end{aligned}$$

where

$$E_{\pi^B}\left[Y_{n+1}^{WAR}(a_F, k)\right] = \pi^B Y_{n+1}^{WAR}(a_F, k+1) + \left(1 - \pi^B\right) Y_{n+1}^{WAR}(a_F, k).$$